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Kinetic Integrated Modeling of Heating and Current Drive in Tokamaks

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Outline

- **Integrated Simulation of Tokamak Plasmas**
- **Integrated Tokamak Modeling Code: TASK**
- **Beam Tracing Analysis**: TASK/WR
- **2D FEM Full Wave Analysis**: TASK/WF
- **Integral Form of Dielectric tensor**: TASK/DP
- **Kinetic integrated modeling**: TASK/FP
- **Summary**

Integrated Simulation of Burning Plasmas

In order to

- predict the performance of future fusion devices,
- optimize their operation scenario,
- contribute to acceptable design of DEMO reactor

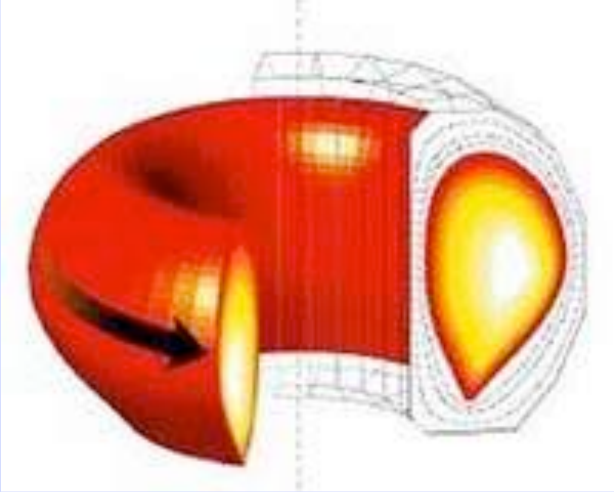
We need a reliable tool to describe

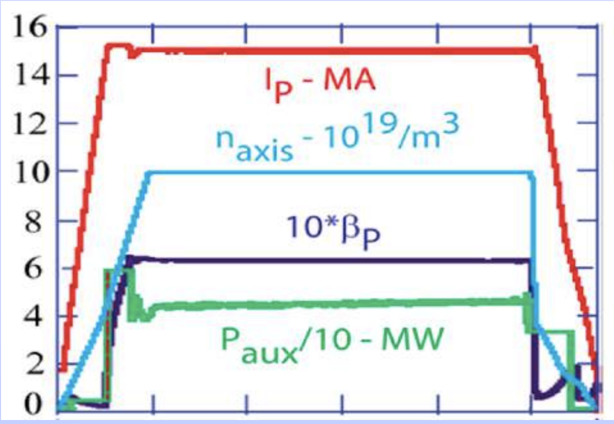
Whole plasma

- core, edge, scrape-off layer, divertor plasmas, and plasma-wall interactions

Whole discharge period

- startup, sustainment, probabilistic incidents, and shut down

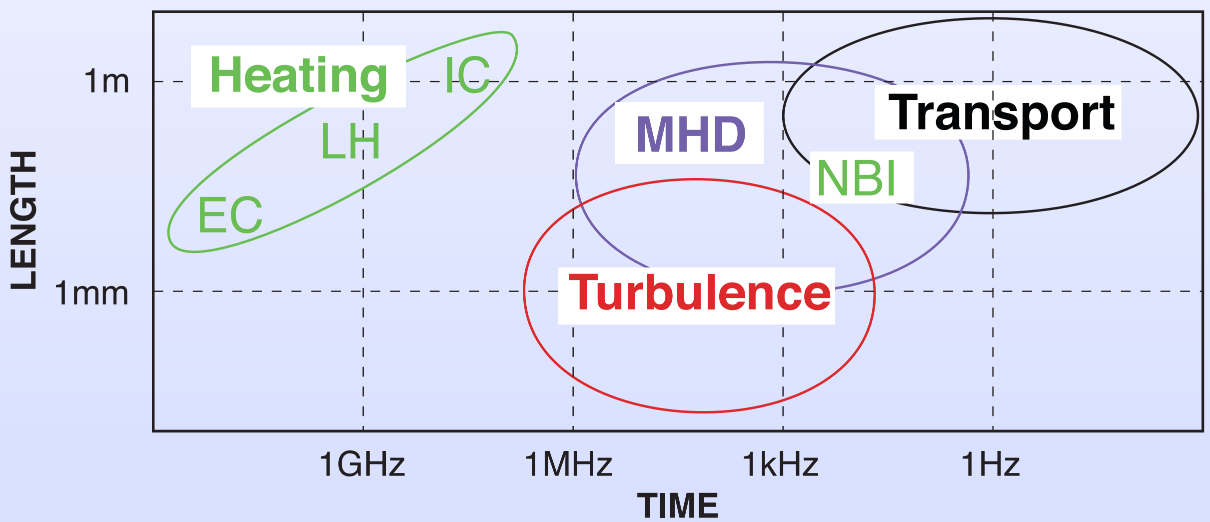




Modeling of Tokamak Plasmas

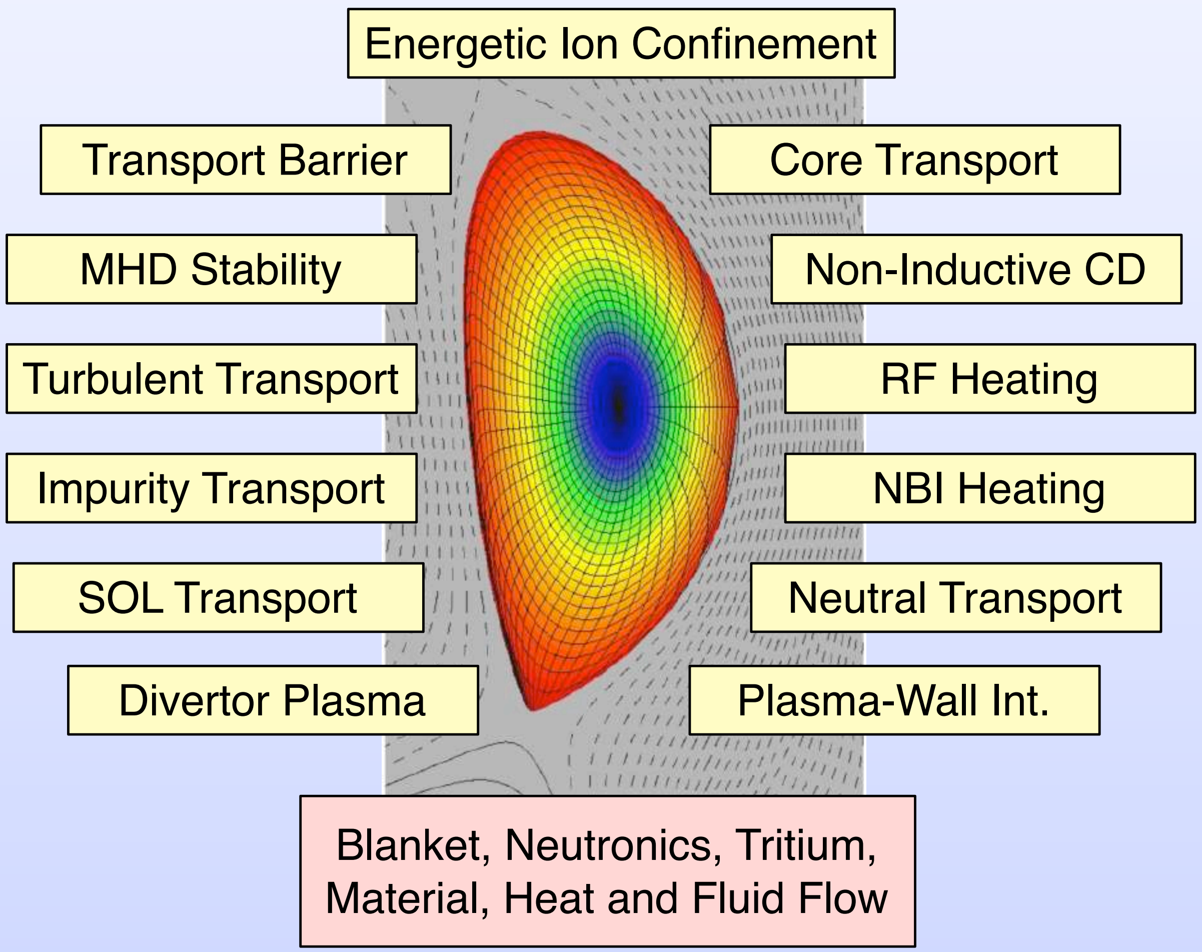
Broad range of time scale
100 GHz 〜 1000 s

Broad range of spatial scale:
10 μm 〜 10 m



One simulation code never coves all range.

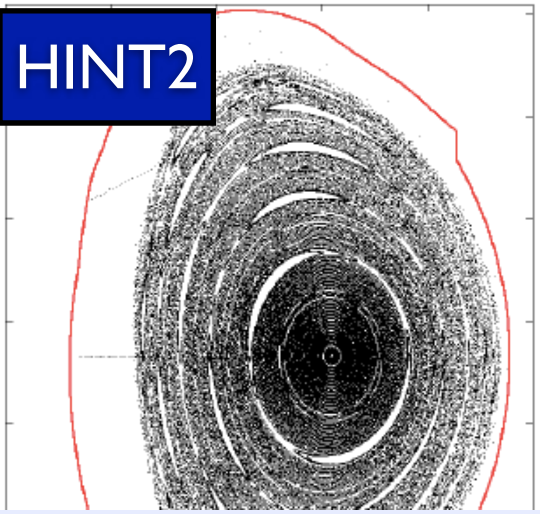
Various phenomena in toroidal plasmas



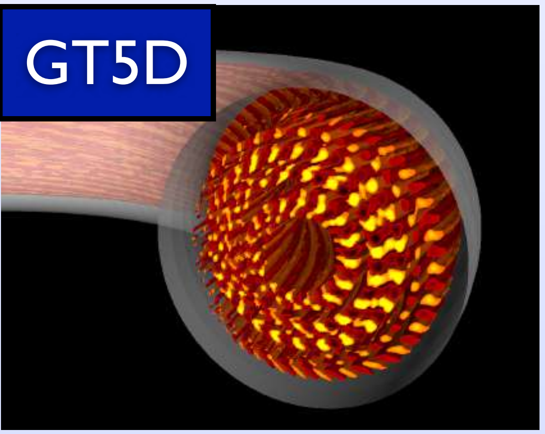
✻ Wide range of time scale, spatial scale, and understanding

- Integrated simulation combining modeling codes
- Various levels of physics model

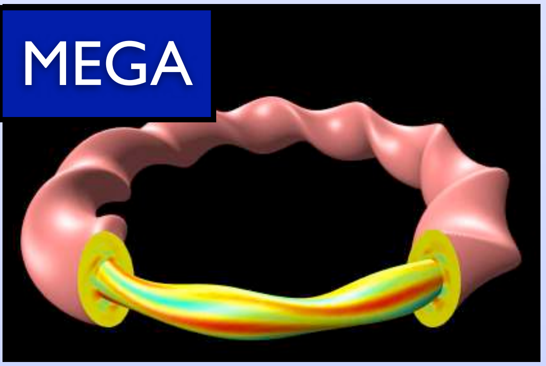
Structure of Integrated Modeling



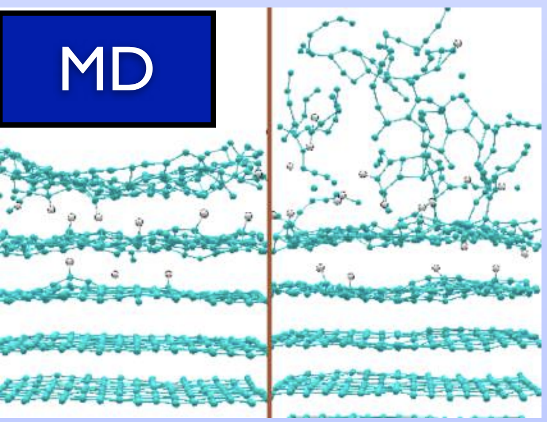
3D MHD Equilibrium



Turbulence Simulation



Nonlinear MHD



Turbulence Simulation

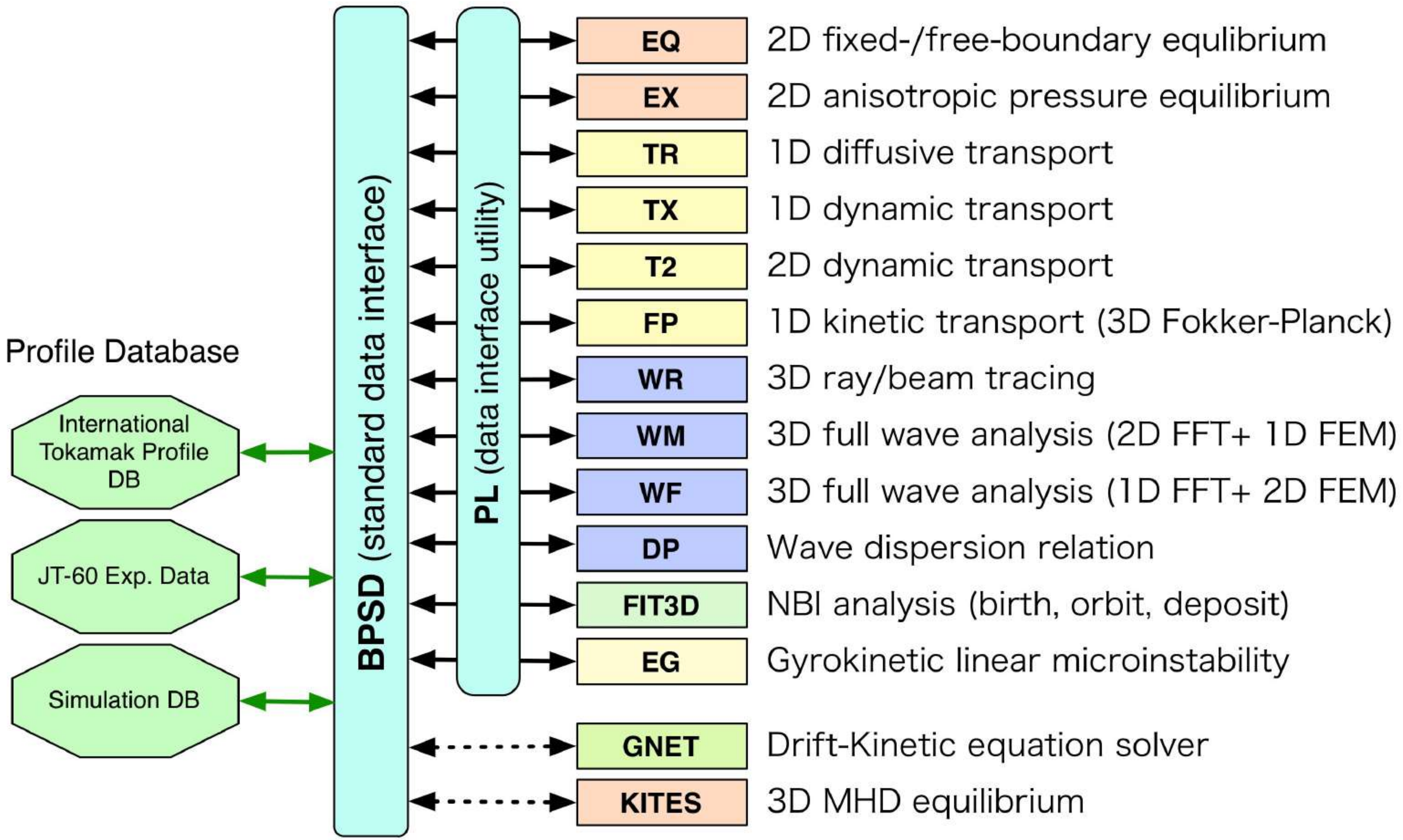
Material Simulation

- **Integrated Modeling**
  - Diagnostic model
  - Control model
  - **Equilibrium:**
    - Rotation, Anisotropy
    - Magnetic Island
    - Fixed boundary
    - Free boundary
    - Time evolution
  - **Core Plasma Transport:**
    - Neoclassical Transport
    - Neutral Transport
    - Turbulent Transport
    - Impurity Transport
    - Radiation Transport
    - Energetic Particles
    - Diffusive transport
    - Dynamic transport
    - Kinetic transport
    - Particle orbit
  - **Global Stability:**
    - Linear Stability
    - Plasma Deformation
    - MHD stability
    - Kinetic stability
  - **Source:**
    - NBI
    - Wave: IC, LH, EC
    - Pellet
    - Ray/beam tracing
    - Full wave
    - Pellet ablation
  - **Peripheral Plasma Transport:**
    - Collisional Transport
    - Neutral Transport
    - Turbulent Transport
    - Impurity Transport
    - Plasma-Wall Interaction
    - Radiation Transport
    - Fluid model
    - Kinetic model
    - Particle model

Integrated Modeling Code: TASK

- **Transport Analysing System for TokamaK**
- **Features**
  - **Modular structure**: easier maintenance and update
  - **Standard data set and interface**: BPSD proposed by BPSI
  - **Multi-level analysis**: Diffusive, dynamic, and kinetic transport
  - **Various Heating and Current Drive Scheme**: EC, LH, IC, AW, NB
  - **High Portability**: Unix-like OS, and X11
  - **Development using CVS**: Version control for collaboration
  - **Open Source**: http://bpsi.nucleng.kyoto-u.ac.jp/task/
  - **Parallel Processing**: MPI and PETSc

Present Structure of TASK



Development since 1992, now in Kyoto University

Analysis of RF Heating and Current Drive

- **Analysis of wave propagation and absorption**
  - **Geometrical optics**: Eikonal approximation
    - **Ray tracing**
    - **Beam tracing**
  - **Full wave analysis**: Boundary-value problem of Maxwell's equation
    - **Fourier transform**
    - **Discrete differential equation**
- **Analysis of velocity distribution function**
  - **Fokker-Planck analysis**: Finite difference/element method
  - **Monte-Carlo analysis**: Orbit following

Geometrical Optics: TASK/WR

- **Ray Tracing**
  - **Time evolution of ray**
    - $\boldsymbol{r}$: ray position
    - $\boldsymbol{k}$: wave number
    - $K(\omega, \boldsymbol{k}; \boldsymbol{r})$: dispersion relation

$$\frac{\mathrm{d}r^\alpha}{\mathrm{d}\tau} = \frac{\partial K}{\partial k_\alpha}$$

$$\frac{\mathrm{d}k_\alpha}{\mathrm{d}\tau} = -\frac{\partial K}{\partial r^\alpha}$$

  - **Condition for geometrical optics**:

$$\delta \equiv \frac{\lambda}{L} \ll 1$$

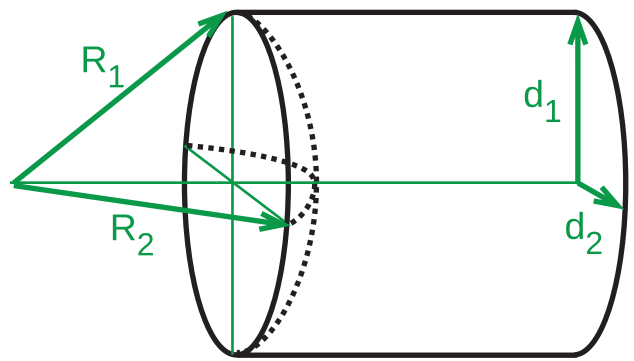
    - λ: wave length
    - L: characteristic scale length of the medium

Finite Beam Size

- **Beam tracing**
  - **Fresnel condition**: diffraction is negligible

$$\frac{d^2}{L\lambda} \gg 1$$

    - Plane wave: Beam size d is sufficiently large
    - Beam : Diffraction effect determines the beam size d
  - **Propagation of beam** with
    - finite beam radius
    - finite curvature radius



  - **References**
    - G. V. Pereverzev, in *Reviews of Plasma Physics*, Vol. 19, p. 1.
    - A. G. Peeters, Phys. Plasmas **3** (1996) 4386.
    - G. V. Pereverzev, Phys. Plasmas **4** (1998) 3529.

Beam Tracing Method

- **Beam shape** : Weber function Hermite polynomial: H_n)

$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{Re}\left[\sum_{mn} C_{mn}(\delta^2\boldsymbol{r})\boldsymbol{e}(\delta^2\boldsymbol{r})H_m(\delta\xi_1)H_n(\delta\xi_2)\,\mathrm{e}^{\,\mathrm{i}\,s(\boldsymbol{r})-\phi(\boldsymbol{r})}\right]$$

  - Amplitude : C_{mn}
  - Polarization : $\boldsymbol{e}$
  - Phase : $s(\boldsymbol{r}) + \mathrm{i}\,\phi(\boldsymbol{r})$

$$s(\boldsymbol{r}) \;=\; s_0(\tau) + k^0_\alpha(\tau)[r^\alpha - r^\alpha_0(\tau)] + \frac{1}{2}s_{\alpha\beta}[r^\alpha - r^\alpha_0(\tau)][r^\beta - r^\beta_0(\tau)]$$

$$\phi(\tau) \;=\; \frac{1}{2}\phi_{\alpha\beta}[r^\alpha - r^\alpha_0(\tau)][r^\beta - r^\beta_0(\tau)]$$

  - Position of beam axis : $\boldsymbol{r}_0$, Wave number on beam axis: $\boldsymbol{k}^0$
  - **Curvature radius** of equi-phase surface: $R_\alpha = 1/\lambda s_{\alpha\alpha}$
  - **Beam radius**: $d_\alpha = \sqrt{2/\phi_{\alpha\alpha}}$

Beam Propagation Equation

- **Solvable condition** for Maxwell's equation with beam field

$$\frac{\mathrm{d}r_0^\alpha}{\mathrm{d}\tau} = \frac{\partial K}{\partial k_\alpha}$$

$$\frac{\mathrm{d}k_\alpha^0}{\mathrm{d}\tau} = -\frac{\partial K}{\partial r^\alpha}$$

$$\frac{\mathrm{d}s_{\alpha\beta}}{\mathrm{d}\tau} = -\frac{\partial^2 K}{\partial r^\alpha \partial r^\beta} - \frac{\partial^2 K}{\partial r^\beta \partial k_\gamma} s_{\alpha\gamma} - \frac{\partial^2 K}{\partial r^\alpha \partial k_\gamma} s_{\beta\gamma} - \frac{\partial^2 K}{\partial k_\gamma \partial k_\delta} s_{\alpha\gamma} s_{\beta\delta} + \frac{\partial^2 K}{\partial k_\gamma \partial k_\delta} \phi_{\alpha\gamma} \phi_{\beta\delta}$$

$$\frac{\mathrm{d}\phi_{\alpha\beta}}{\mathrm{d}\tau} = -\left(\frac{\partial^2 K}{\partial r^\alpha \partial k_\gamma} + \frac{\partial^2 K}{\partial k_\gamma \partial k_\delta} s_{\alpha\delta}\right) \phi_{\beta\gamma} - \left(\frac{\partial^2 K}{\partial r^\beta \partial k_\gamma} + \frac{\partial^2 K}{\partial k_\gamma \partial k_\delta} s_{\beta\delta}\right) \phi_{\alpha\gamma}$$

  - 18 ordinary differential equations

- **Equation for the wave amplitude**: C_{mn}

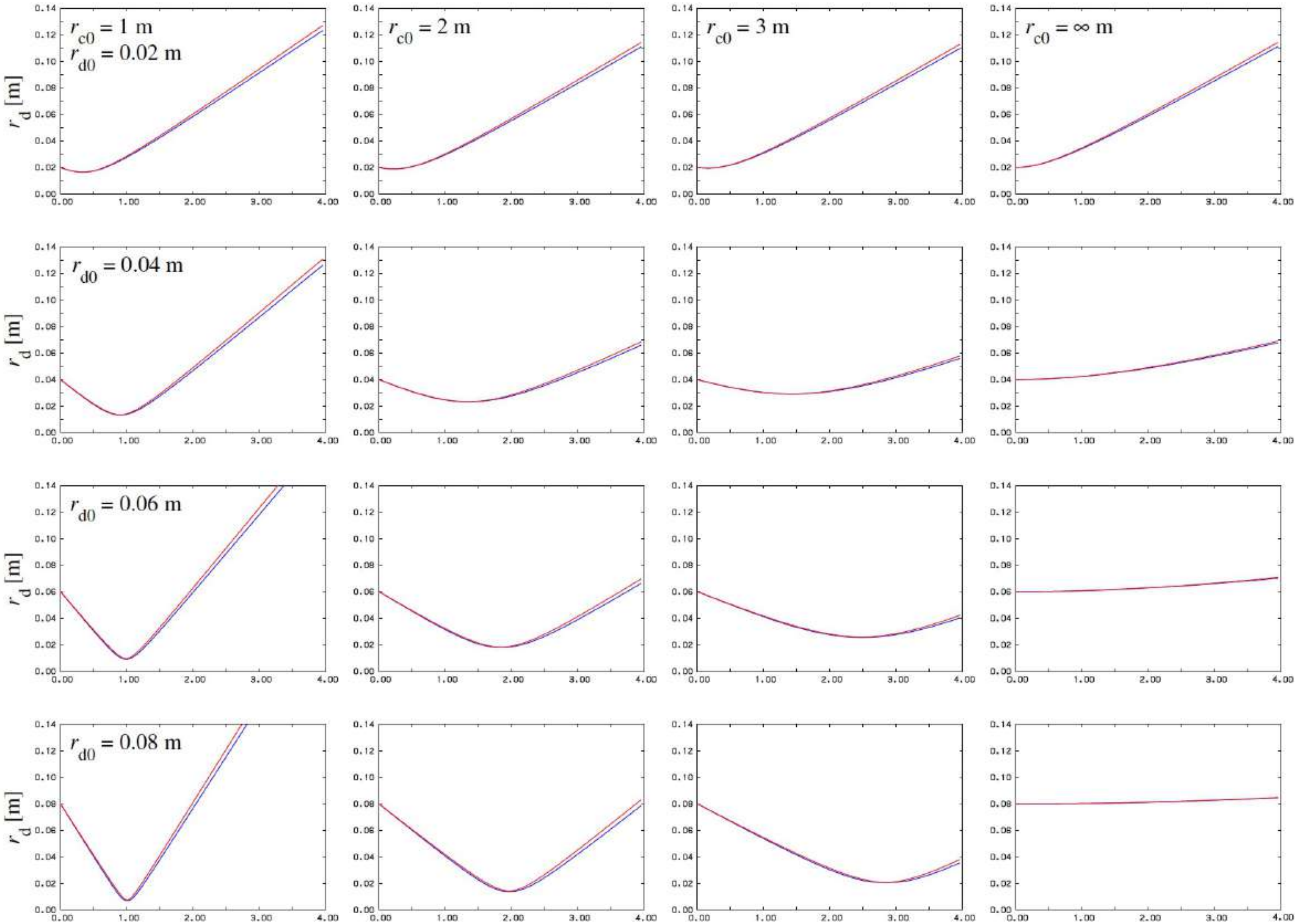
$$\nabla \cdot \left(\boldsymbol{v}_{\mathrm{g}0} |C_{mn}|^2\right) = -2\gamma |C_{mn}|^2$$

  - Group velocity: $\boldsymbol{v}_{\mathrm{g}0}$
  - Damping rate: $\gamma \equiv (\boldsymbol{e}^* \cdot \overleftrightarrow{\epsilon}_{\mathrm{A}} \cdot \boldsymbol{e})/(\partial K/\partial \omega)$

Defraction of wave beam in vacuum

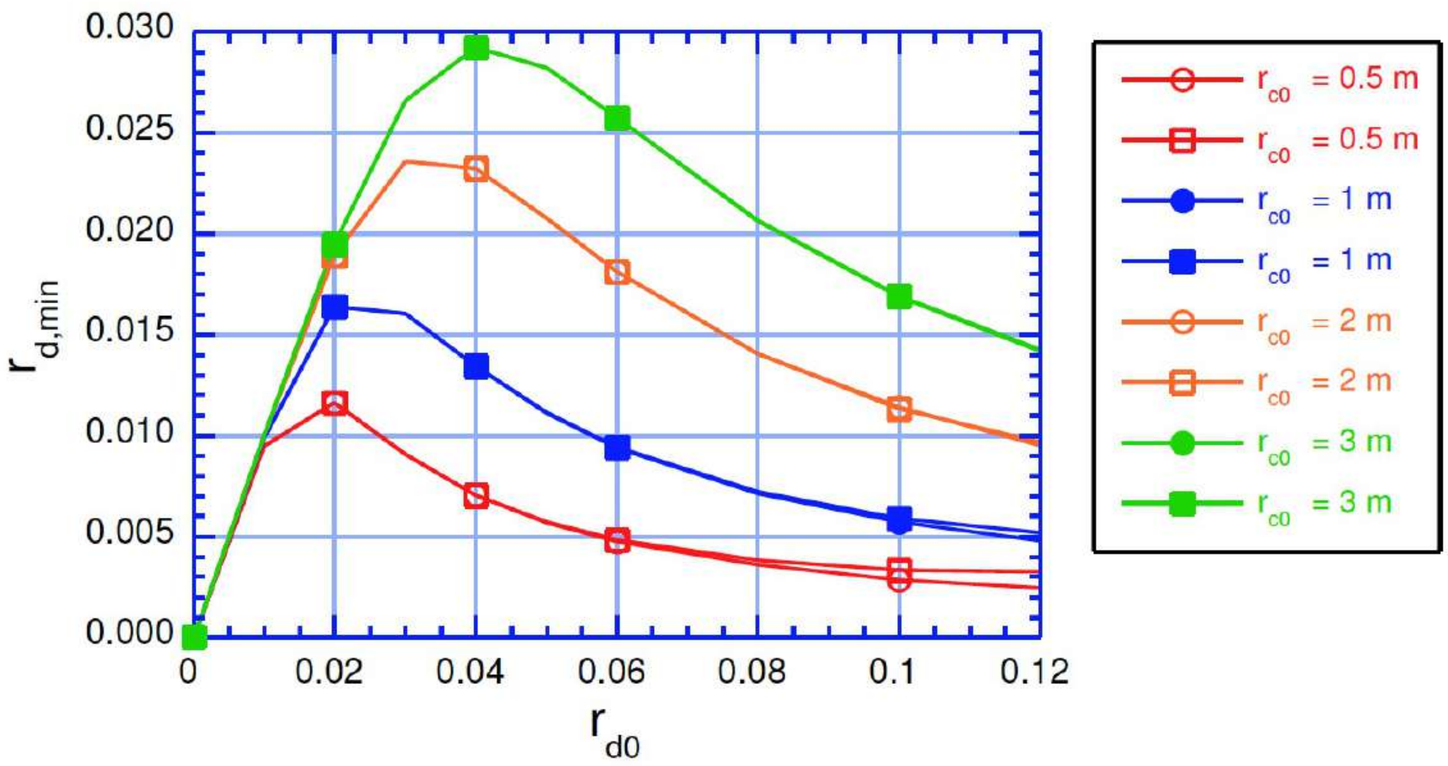
- **Beam radius as a function of path length**: $f = 170\,\mathrm{GHz}$, $\lambda = 1.76\,\mathrm{mm}$

$d_0 \setminus R_0$	1 m	2 m	3 m	∞ m
20 mm				
40 mm				
60 mm				
80 mm				



Dependence of Minimum Beam Radius

- Dependence of minimum beam radius $r_{\mathrm{d,min}}$ on
  - initial beam radius: r_{d0}
  - initial curvature radius: r_{c0}

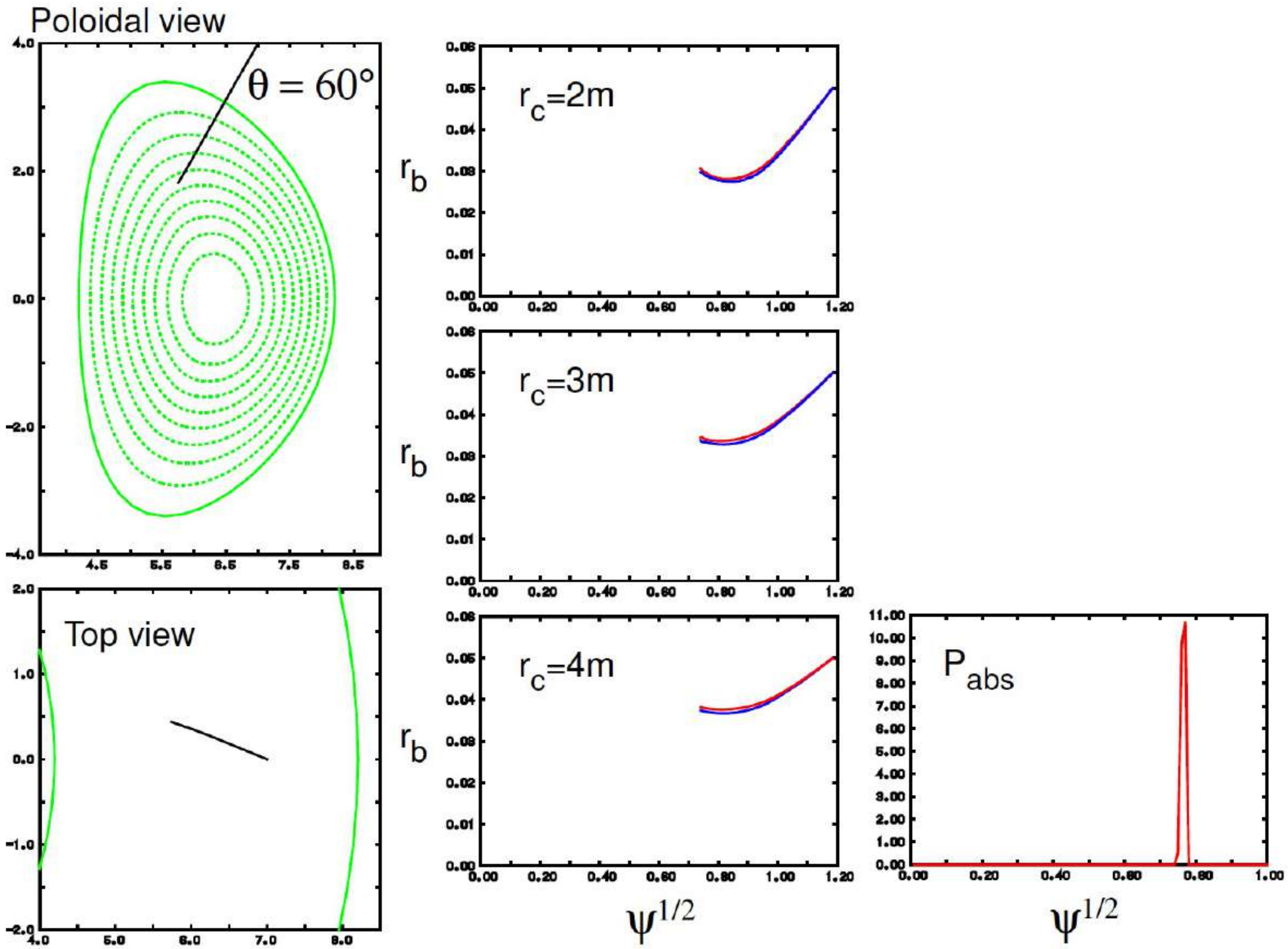


- **Scaling**

$$r_{\mathrm{d,min}} \sim \frac{\lambda}{\pi r_{\mathrm{d0}}} r_{\mathrm{c0}}$$

Application to ITER EC beam

- $f = 170\,\mathrm{GHz}$, toroidal angle $20°$, poloidal angle $60°$
- Initial beam radius: $50\,\mathrm{mm}$



Full Wave Analysis

- **Boundary-value problem of Maxwell's equation with fixed ω**
  - $\boldsymbol{E}$: wave electric field
  - $\overleftrightarrow{\epsilon}$: dielectric tensor

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- **Merit of full wave analysis**
  - Wave length longer than the scale length of medium
  - Propagation over an evanescent layer
  - Coupling to antenna
  - Formation of standing wave
- **Method of full wave analysis**
  - **Fourier analysis**: algebraic equation
  - **Discrete differential equation**: finite difference/element method
  - **Mixture of above two methods**

Kinetic Full Wave Analysis: TASK/WM

- **Magnetic surface coordinate**: (ψ, θ, φ)
- **Wave electric field** in local coordinates: $\boldsymbol{E} = (E_+, E_-, E_{\|})$
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\, \omega \mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

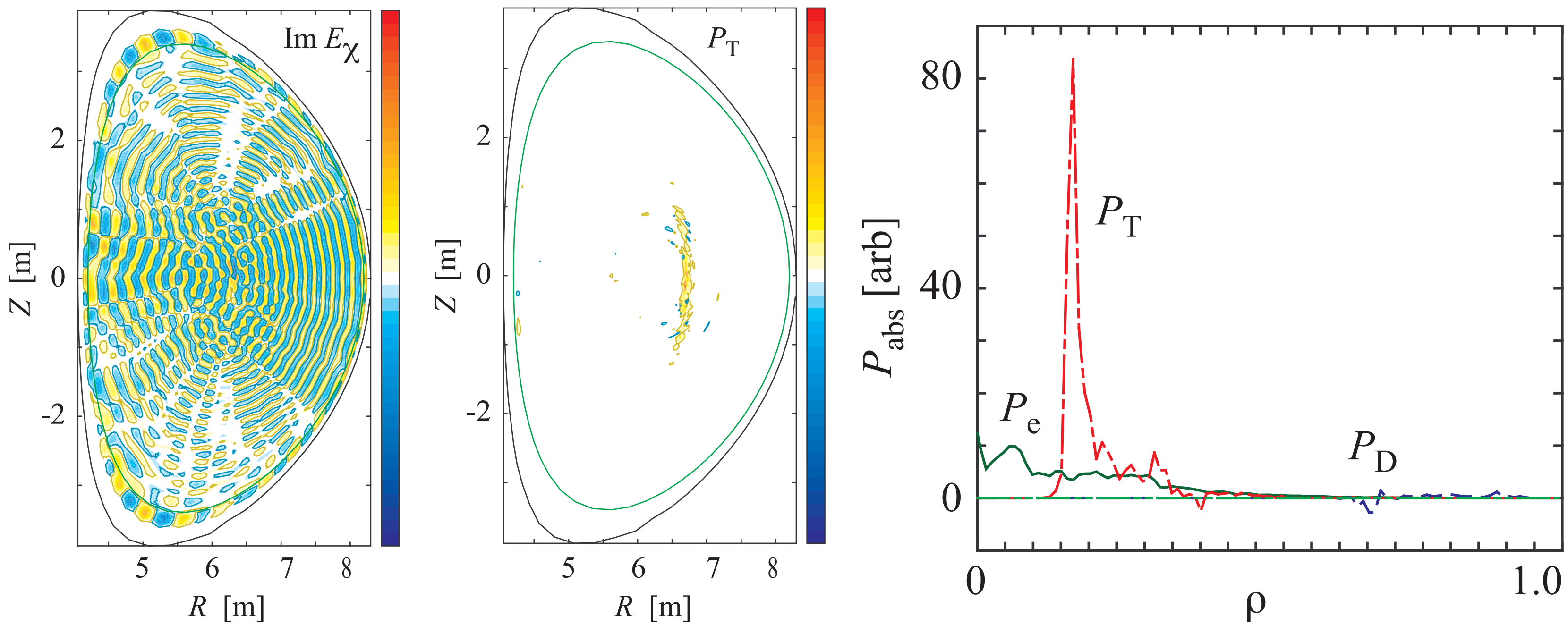
- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$ TASK/DP
  - Wave-particle resonance: Maxwellian, arbitrary $f(v)$
  - FLR effect: Fast wave approximation
- Poloidal and toroidal direction: **mode expansion**
- Radial direction: **FEM**
- Eigenmode analysis: **Complex eigen frequency**

ICRF Heating: TASK/WM

- **Second-harmonic heating of T**: $f = 55\,\mathrm{MHz}$, $n_\phi = 18$
- Power partition

Species	e	D	T	^{4}He
$n(0)$ $[10^{20}\,\mathrm{m}^3]$	1.0	0.5	0.5	0.0
P_{abs} [%]	36.0	1.5	60.7	0.0

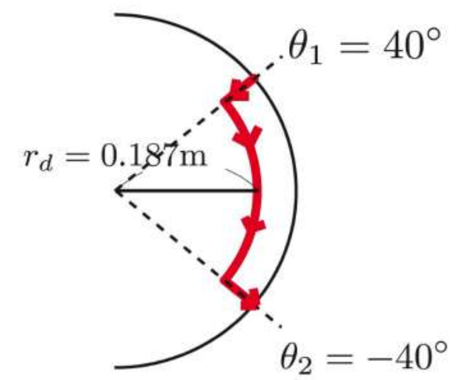
$\mathrm{Im}(E_{\mathrm{th}})$ P_{He3} $P_{\mathrm{abs}}(r)$



Full wave analysis by FEM: TASK/WF

- **Wave electric field with complex frequency**: $\tilde{\boldsymbol{E}}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\,\omega t}$
- **Maxwell's equation**: $\nabla \times \nabla \times \boldsymbol{E} - \dfrac{\omega^2}{c^2}\overleftrightarrow{\epsilon} \cdot \boldsymbol{E} = \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$
  - $\overleftrightarrow{\epsilon}$: Dielectric tensor
    - Collisional cold plasma model
- **Numerical method**: FEM
  - **3D version**
    - Tetrahedron element
    - Electric field along a edge of a tetrahedron
  - **2D version**: axisymmetric cylindrical
    - Triangular element
    - Scalar (toroidal) and vector (poloidal) hybrid basis function
- **Matrix solver**: performance better than TASK/WM expected

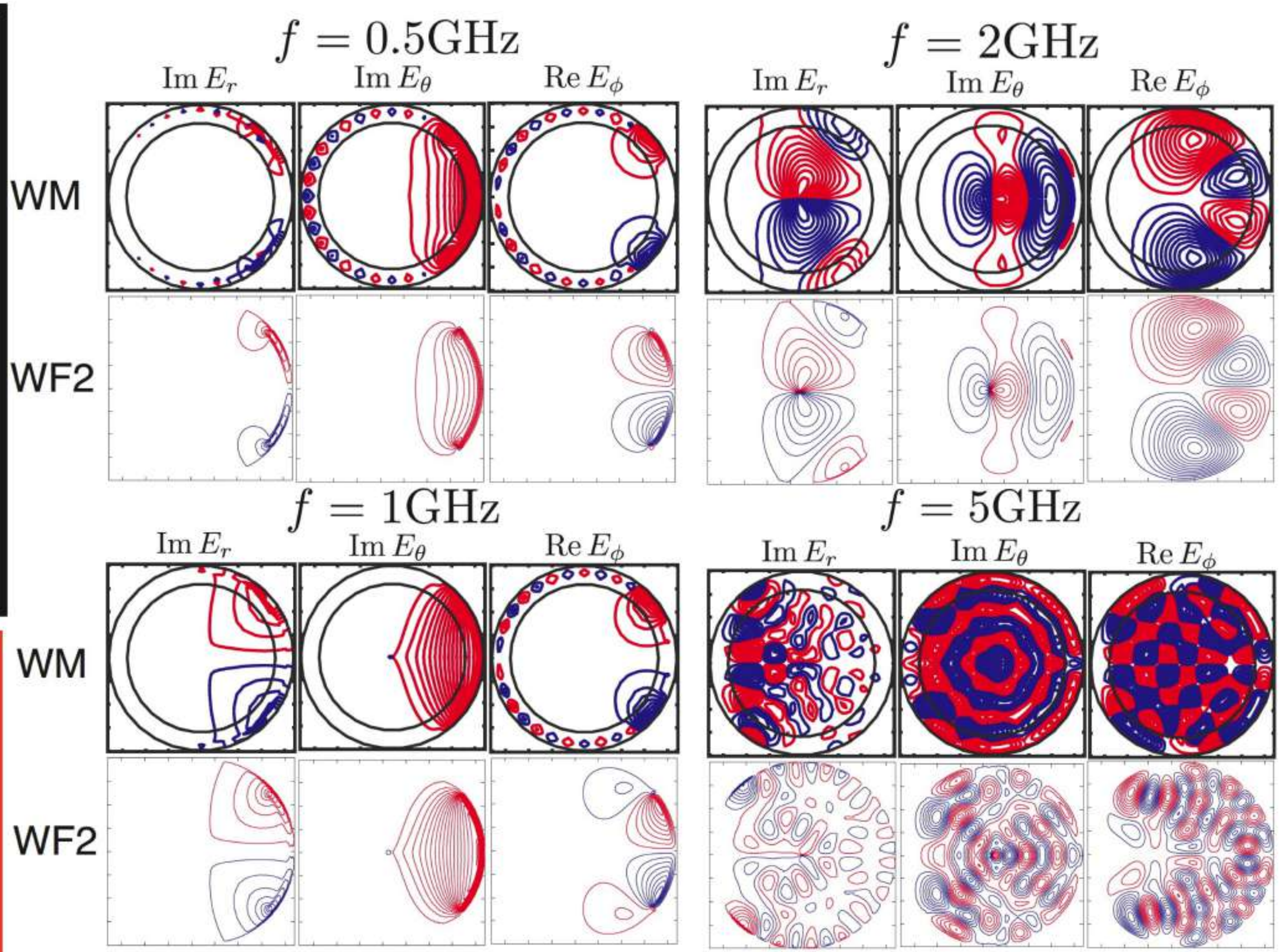
Benchmark test with TASK/WM in Vacuum



major radius	R	0.22m
minor radius	a	0.16m
toroidal mode number	n_ϕ	8

Resolution

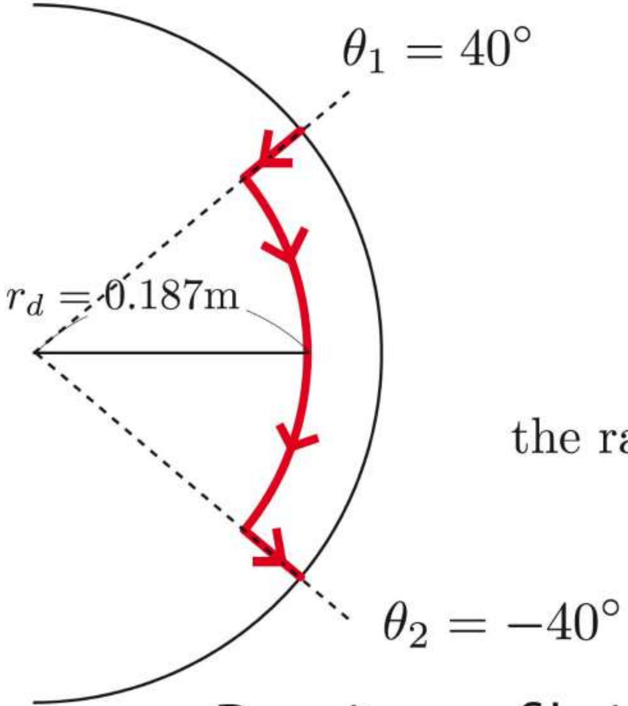
- WF2: 25350 elements
- WM:
  - radial direction: 100 points
  - poroidal direction: 32 modes



- TASK/WM and WF2 have different spacial resolution, so there are some differences in their results.

EC waves in a spherical tokamak

- Antenna



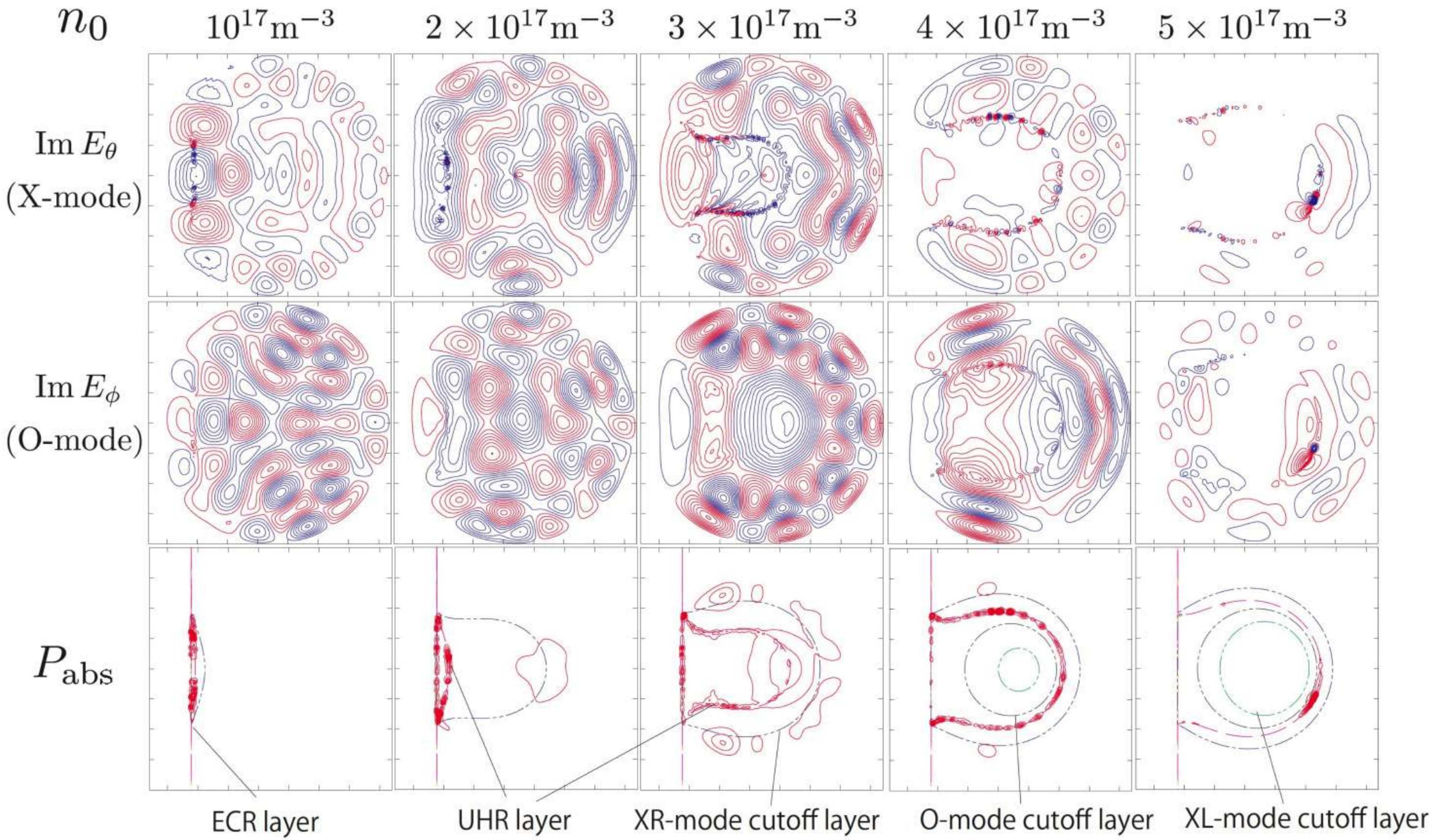
- Used parameters (**LATE**: Kyoto University)

major radius	R	0.22m
minor radius	a	0.16m
wave frequency	f	5GHz
the number of elements	NEMAX	25350
the ratio of collision frequency to ω	ν/ω	1.0×10^{-3}

- Density profile is parabolic.
- Collisional cold plasma model is used for the dielectric tensor.
  - Mode conversion of EC wave into EB wave does not occur.

Density Dependence ($n_\phi = 8$)

- $B_0 = 0.072$T



Dielectric tensor: TASK/DP

- General form of dielectric tensor

$$\nabla\times\nabla\times\boldsymbol{E}(\boldsymbol{r},\omega)-\frac{\omega^2}{c^2}\int_V \mathrm{d}\boldsymbol{r}'\,\overleftrightarrow{\epsilon}(\boldsymbol{r},\boldsymbol{r}';\omega)\cdot\boldsymbol{E}(\boldsymbol{r}',\omega)-\mathrm{i}\,\omega\mu_0\boldsymbol{J}_{\mathrm{ext}}(\boldsymbol{r},\omega)=\boldsymbol{0}$$

- **Various models of dielectric tensor**
  - **MHD plasma model** (resistive)
  - **Cold plasma model** (collisional)
  - **Hot plasma model** in a uniform plasma (Fourier expansion)
    - **Plasma dispersion function**: parallel wave-particle interaction
    - **Modified Bessel functions**: finite gyro radius effects
    - **Numerical integration in velocity space**: arbitrary $f(v_{||},v_\perp)$
  - **Differential form** in an inhomogeneous plasma (up to $(k_\perp\rho)^2$)
  - **Integral form** in an inhomogeneous plasma (applicable to $k_\perp\rho > 1$)

Differential Form of Dielectric Tensor

- **Finite gyro radius effects up to second order**

$$\overleftrightarrow{\epsilon}\cdot\boldsymbol{E}(\boldsymbol{r}) = \left[\overleftrightarrow{\epsilon}^{(0)}-\frac{\mathrm{i}}{2}\left(\frac{\partial\overleftrightarrow{\epsilon}^{(1)}}{\partial x}\right)-\frac{1}{8}\left(\frac{\partial^2\overleftrightarrow{\epsilon}^{(2)}}{\partial x^2}\right)-\mathrm{i}\overleftrightarrow{\epsilon}^{(1)}\frac{\partial}{\partial x}-\frac{1}{2}\frac{\partial}{\partial x}\overleftrightarrow{\epsilon}^{(2)}\frac{\partial}{\partial x}\right]\cdot\boldsymbol{E}(\boldsymbol{r})$$

whereC

$$\overleftrightarrow{\epsilon}(k_\perp) = \overleftrightarrow{\epsilon}^{(0)} + \overleftrightarrow{\epsilon}^{(1)}k_\perp + \overleftrightarrow{\epsilon}^{(2)}k_\perp^2/2$$

  – Applicable of $k_\perp\rho_s \ll 1$
  – Second harmonic cyclotron damping and TTMP (Landau damping due to magnetic field) can be described.
  – Since the second order derivatives are included in Maxwell's equation, the formulation of numerical scheme is relatively simple.
  – Absorption power

$$P_s = \frac{1}{2}\left[\boldsymbol{E}^*\cdot\overleftrightarrow{\sigma}_s^{(0)}\cdot\boldsymbol{E}-\frac{\mathrm{i}}{2}\left(\boldsymbol{E}^*\cdot\overleftrightarrow{\sigma}_s^{(1)}\cdot\frac{\partial\boldsymbol{E}}{\partial x}-\frac{\partial\boldsymbol{E}^*}{\partial x}\cdot\overleftrightarrow{\sigma}_s^{(1)}\cdot\boldsymbol{E}\right)\right.$$
$$\left.+\frac{1}{2}\frac{\partial\boldsymbol{E}^*}{\partial x}\cdot\overleftrightarrow{\sigma}_s^{(2)}\cdot\frac{\partial\boldsymbol{E}}{\partial x}+\frac{1}{8}\left(\boldsymbol{E}^*\cdot\frac{\partial\overleftrightarrow{\sigma}_{\mathrm{sH}}^{(2)}}{\partial x}\cdot\frac{\partial\boldsymbol{E}}{\partial x}+\frac{\partial\boldsymbol{E}^*}{\partial x}\cdot\frac{\partial\overleftrightarrow{\sigma}_{\mathrm{sH}}^{(2)}}{\partial x}\cdot\boldsymbol{E}\right)\right]$$

Necessity of Integral Formulation of Dielectric Tensor

- **Wave-particle resonance interaction in an inhomogeneous plasma**
  - Landau damping
  - Cyclotron damping
- **Finite gyro radius effects in higher order**
  - Cyclotron harmonic damping
  - Bernstein waves
  - Interaction with energetic particles
- **Merits of integral formulation compared with spectral method (FFT)**
  - Spatially localized interaction
  - Sparse coefficient matrix in the matrix equation
  - Efficient for parallel processing for large scale calculation

Integral Formulation of Wave-Particle Interaction (1)

- **Particle orbit**:

$$\boldsymbol{r} = \boldsymbol{r}' + \Delta\boldsymbol{r}(\boldsymbol{v}, t - t')$$

$$\boldsymbol{v} = \boldsymbol{v}' + \Delta\boldsymbol{v}(\boldsymbol{r} - \boldsymbol{r}', t - t')$$

  where $\Delta\boldsymbol{r} = \Delta\boldsymbol{v} = \boldsymbol{0}$ for $t = t'$

- A particle with velocity $\boldsymbol{v}$ experiences the electric field $\boldsymbol{E}(\boldsymbol{r}')$ at the position $\boldsymbol{r}'$ where the particle was $t - t'$ ago.

- **Linearized Vlasov equation** for $\boldsymbol{E}(\tilde{\boldsymbol{r}}, t) = \boldsymbol{E}(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\,\omega t}$

$$\frac{\mathrm{d}f(\boldsymbol{r}', \boldsymbol{v}', t')}{\mathrm{d}t'} = -\frac{q}{m}\boldsymbol{E}(\boldsymbol{r}')\cdot\frac{\partial f_0(\boldsymbol{r}', \boldsymbol{v}')}{\partial \boldsymbol{v}'}\,\mathrm{e}^{-\mathrm{i}\,\omega t'}$$

- **Perturbed distribution**:

$$f(\boldsymbol{r}, \boldsymbol{v}, t) = -\frac{q}{m}\int_{-\infty}^{t}\mathrm{d}t'\,\boldsymbol{E}(\boldsymbol{r}')\cdot\frac{\partial f_0(\boldsymbol{r}', \boldsymbol{v}')}{\partial \boldsymbol{v}'}\,\mathrm{e}^{-\mathrm{i}\,\omega t'}$$

Integral Formulation of Wave-Particle Interaction (2)

- **Induced current**:

$$\boldsymbol{j}(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\,\omega t} = \int \mathrm{d}\boldsymbol{v}\, q\boldsymbol{v} f(\boldsymbol{r},\boldsymbol{v},t)$$

- Substituting $f(\boldsymbol{r},\boldsymbol{v},t)$ and replacing $\boldsymbol{v}$ by $\boldsymbol{r}'$, we obtain

$$\boldsymbol{j}(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{r}'\, \overleftrightarrow{\sigma}(\boldsymbol{r}-\boldsymbol{r}', t-t')\cdot \boldsymbol{E}(\boldsymbol{r}')$$

- **The integral form of the conductivity tensor** is defined by

$$\overleftrightarrow{\sigma}(\boldsymbol{r}-\boldsymbol{r}', t-t') = -\frac{q}{m}\int_{-\infty}^{t} \mathrm{d}t'\, \boldsymbol{v}\, \frac{\partial f_0(\boldsymbol{r}',\boldsymbol{v}')}{\partial \boldsymbol{v}'}\bigg|_{\substack{\boldsymbol{r}' = \boldsymbol{r} + \Delta\boldsymbol{r}(\boldsymbol{v}, t-t') \\ \boldsymbol{v}' = \boldsymbol{v} - \Delta\boldsymbol{v}(\boldsymbol{r}-\boldsymbol{r}', t-t')}}$$

Full Wave Analysis in an Inhomogeneous Plasma (1)

- **Absorption of plasma wave excited by obliquely incident electro-magnetic wave**
  - **Density profile**:
$$n(z) = n_0\, \mathrm{e}^{-\kappa z}$$
  - **Electrostatic potential** to sustain density gradient
$$\Phi(z) = \frac{\kappa T}{q} z$$
  - **Particle motion**: acceleration by static electric field
$$v_z(t') = v_z + \alpha(t' - t)$$
$$z(t') = z + v_z(t' - t) + \frac{1}{2}\alpha(t' - t)^2$$
$$\alpha = -qE/m = -\kappa T/m$$

Full Wave Analysis in an Inhomogeneous Plasma (2)

- **Maxwell's equation**:

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\, \omega \mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- **Basic equation**: Length is normalized by v_{th}/ω where $v_{\mathrm{th}} = \sqrt{T/m}$

$$\frac{1}{\beta^2} \nabla \times \nabla \times \boldsymbol{E}(z) - \int_{-\infty}^{\infty} \mathrm{d}z' \, \overleftrightarrow{\epsilon}(z - z') \, \boldsymbol{E}(z') = 0$$

- **Integral form of dielectric tensor**

$$\overleftrightarrow{\epsilon}(z - z') = \delta(z - z') \overleftrightarrow{I} + \mathrm{i}\, \frac{\omega_{\mathrm{p0}}^2}{\omega^2} \mathrm{e}^{-\kappa(z+z')}$$

$$\times \begin{pmatrix} U_0 & 0 & 0 \\ 0 & U_0 - n_y^2 \beta^2 U_2 & \mathrm{i}\, n_y \beta[(z - z')U_0 - \kappa U_2] \\ 0 & \mathrm{i}\, n_y \beta[(z - z')U_0 + \kappa U_2] & (z - z')^2 U_{-2} - \kappa^2 U_2 \end{pmatrix}$$

$$\beta = v_{\mathrm{th}}/c, \qquad n_y = k_y c/\omega, \qquad U_n = U_n\left(z - z',\ \sqrt{\kappa^2 + \beta^2 n_y^2}\right)$$

Full Wave Analysis in an Inhomogeneous Plasma (3)

- **Kernel function**

$$U_n(\xi,\eta) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \mathrm{d}\tau\, \tau^{n-1} \exp\left[-\frac{1}{2}\frac{\xi^2}{\tau^2} - \frac{1}{2}\eta^2\tau^2 + \mathrm{i}\,\tau \right]$$

- **Properties of U_n**
  - **Derivative**:

$$\frac{\partial U_n}{\partial \xi} = U_{n-2}$$

  - **Partial integral with respect to τ**

$$nU_{n-1} + \frac{\partial^2}{\partial \xi^2} U_{n+1} - \eta^2 U_{n+1} + \mathrm{i}\, U_n$$

$$= \left[\frac{1}{\sqrt{2}} \tau^{n-1} \exp\left[-\frac{1}{2}\frac{\xi^2}{\tau^2} - \frac{1}{2}\eta^2\tau^2 + \mathrm{i}\,\tau \right] \right]_0^\infty = \begin{cases} -\delta_{n,1} & \text{for } n > 0 \\ -\delta(\xi) & \text{for } n = 0 \end{cases}$$

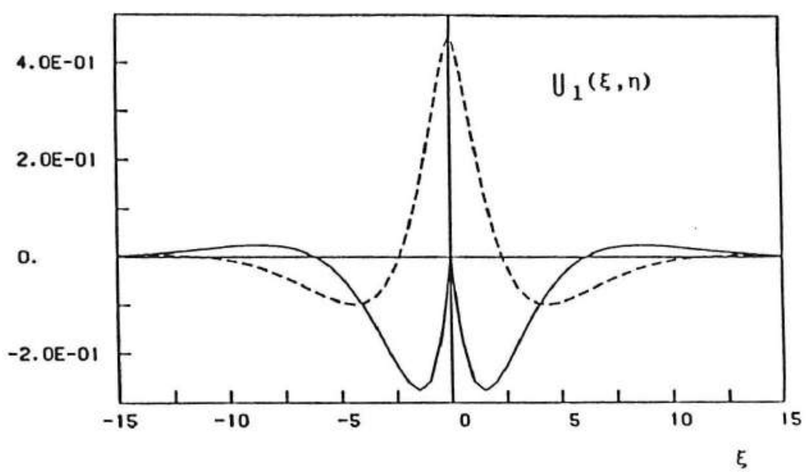
Full Wave Analysis in an Inhomogeneous Plasma (4)

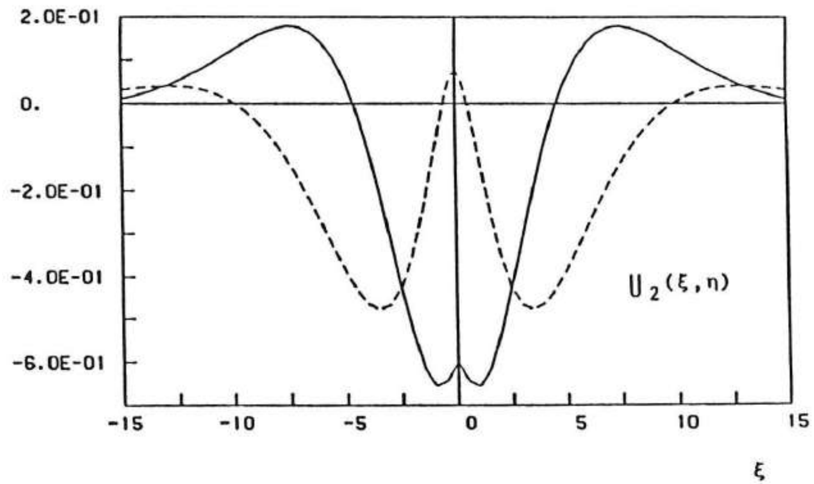
- Using the properties of U_n, the dielectric tensor can be rewritten as

$$\overleftrightarrow{\epsilon}(z-z') = \delta(z-z')\overleftrightarrow{I} - \mathrm{i}\,\frac{\omega_{\mathrm{p}0}^2}{\omega^2}\,\mathrm{e}^{-\kappa(z+z')}$$

$$\times \begin{pmatrix} -U_0 & 0 & 0 \\ 0 & \delta(\xi) - \zeta^2(U_1 - \mathrm{i}\,U_2) - U_1\frac{\partial^2}{\partial z \partial z'} & \zeta U_2 \frac{\partial}{\partial z'} \\ 0 & -\zeta U_2 \frac{\partial}{\partial z} & \delta(\xi) - \zeta^2 U_1 - (U_1 - \mathrm{i}\,U_2)\frac{\partial^2}{\partial z \partial z'} \end{pmatrix}$$

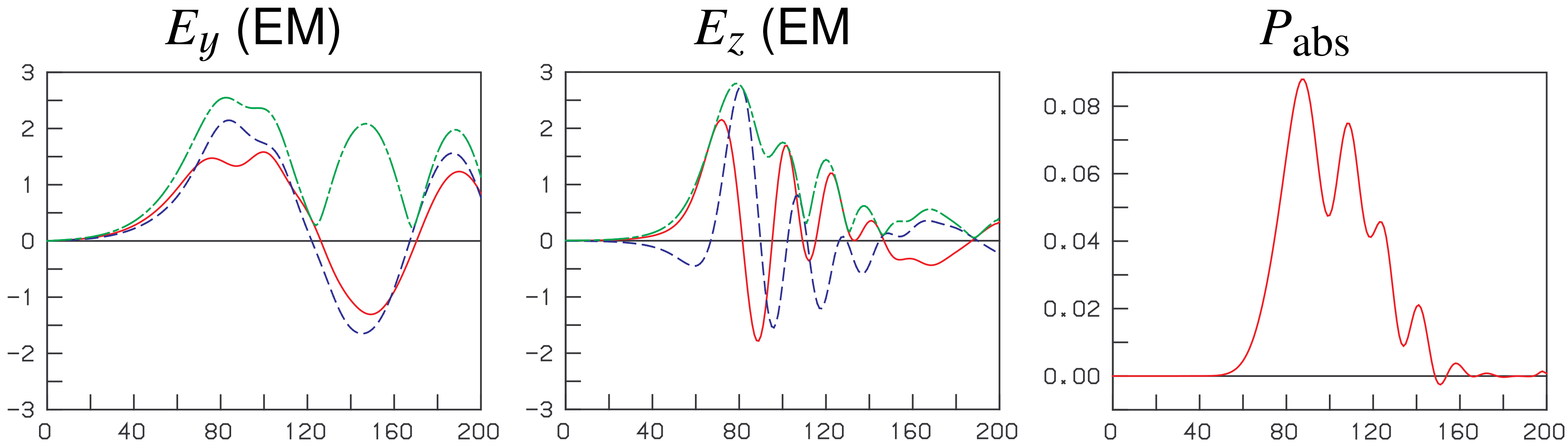
$U_1(\xi, 0)$ $U_2(\xi, 0)$





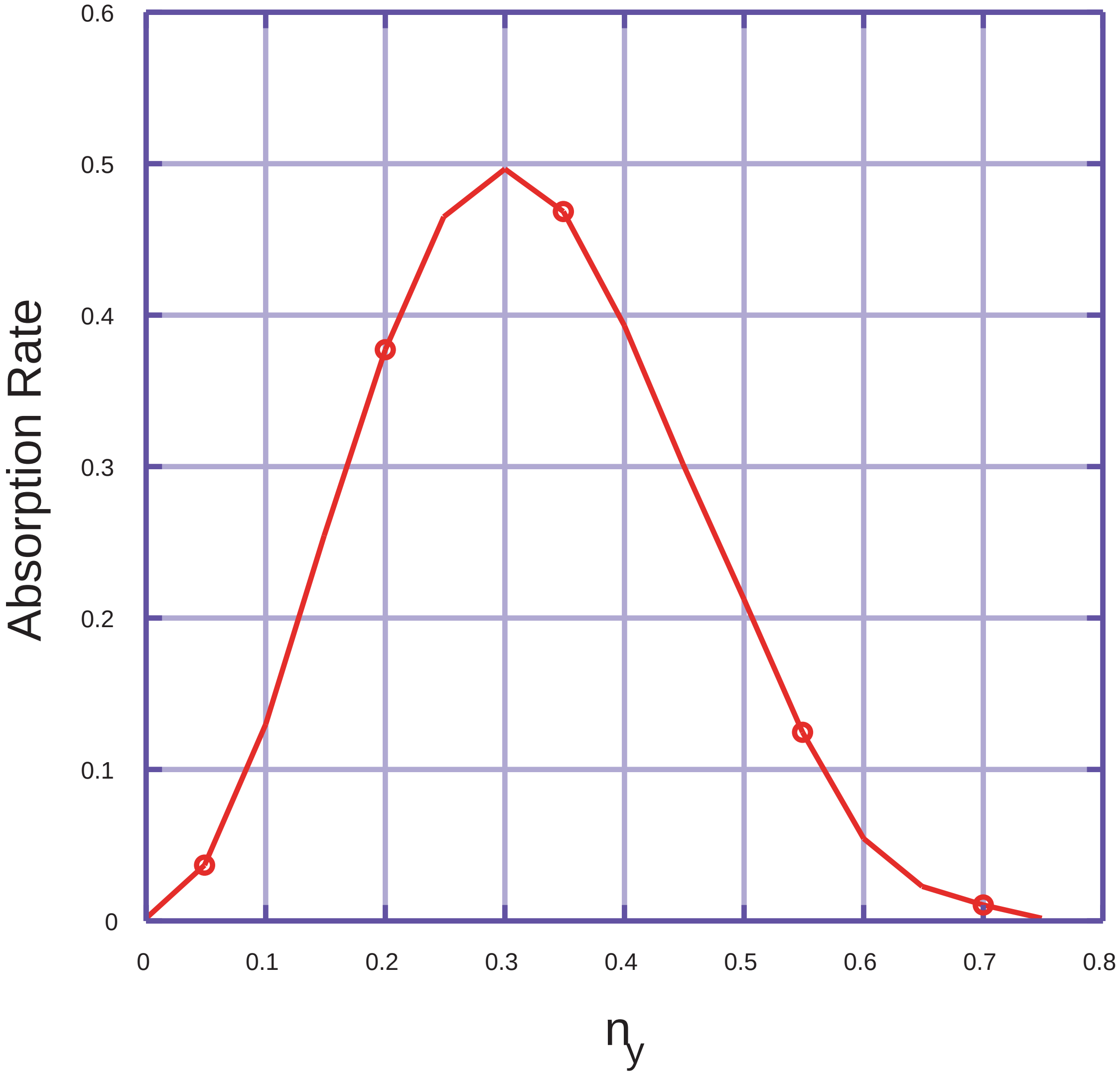
Numerical Result

- **One-dimensional FEM analysis**
- **Density profile**: $(\omega_{\mathrm{p}}^2/\omega^2) = 2\,\mathrm{e}^{-0.005z}$
- **Incident angle**: $n_y = 0.2$
- **Temperature**: $\beta = 0.1$



- **Absorption rate**: 37.7%

Incident Angle Dependence of Absorption Rate



Full Wave Analysis in Magnetized Plasma (1)

- **Absorption at electron cyclotron resonance (ECR)**
  - **Inhomogeneous magnetic field**

$$B_z(z) = B_0\left(1+\frac{z}{L}\right)$$

- **Basic equation**F

$$\frac{1}{\beta^2}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E}(z) - \int_{-\infty}^{\infty}\mathrm{d}z'\,\overleftrightarrow{\epsilon}(z,z')\,\boldsymbol{E}(z') = 0$$

where

$$\overleftrightarrow{\epsilon}(z,z') = \delta(z-z')\overleftrightarrow{I} + \frac{\omega_{\mathrm{p}0}^2}{\omega^2}\begin{pmatrix} (\chi_+\chi_-)/2 & -\mathrm{i}\,(\chi_-\chi_-)/2 & 0 \\ \mathrm{i}\,(\chi_-\chi_-)/2 & (\chi_+\chi_-)/2 & 0 \\ 0 & 0 & \chi_0 \end{pmatrix}$$

Full Wave Analysis in Magnetized Plasma (2)

- **Kernel functions**

$$\chi_\pm = \frac{(1+\kappa z)^{3/2}(1+\kappa z')^{3/2}}{(1+\kappa(z+z')/2)^2} U_0(\xi_\pm)$$

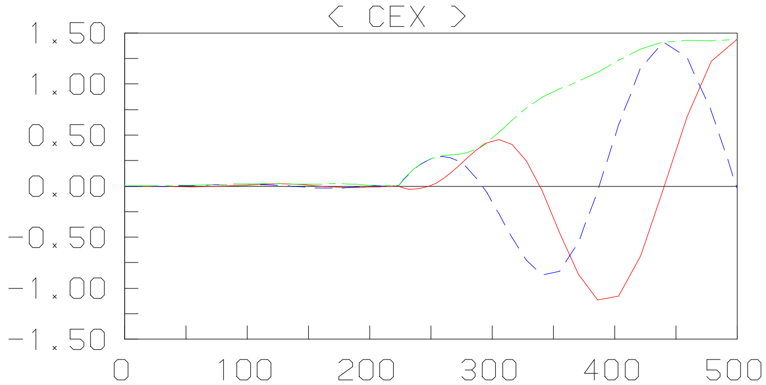
$$\chi_0 = \frac{(1+\kappa z)(1+\kappa z')}{(1+\kappa(z+z')/2)} \left[\xi U_{-2}(\xi) - \frac{\kappa^2}{2(1+\kappa(z+z')/2)^2} U_2(\xi) \right]$$

$$\xi = \frac{\omega(z-z')}{v_{\rm th}}, \qquad \xi_\pm = \frac{(\omega \pm \Omega)(z-z')}{v_{\rm th}}$$

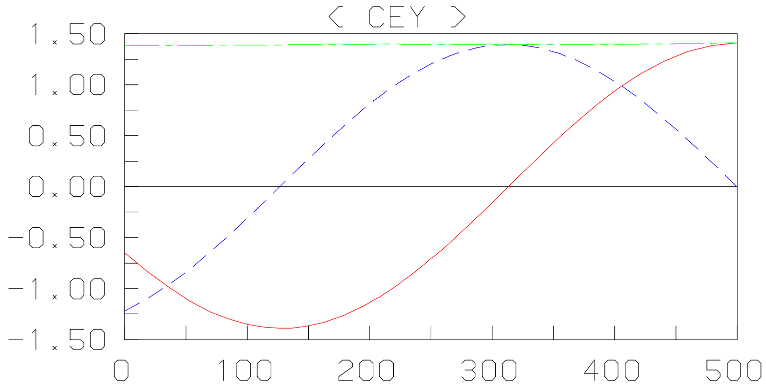
$$\Omega = \frac{qB_0}{m}\left(1 + \frac{z+z'}{2L}\right), \qquad \kappa = \frac{v_{\rm th}}{\omega L}$$

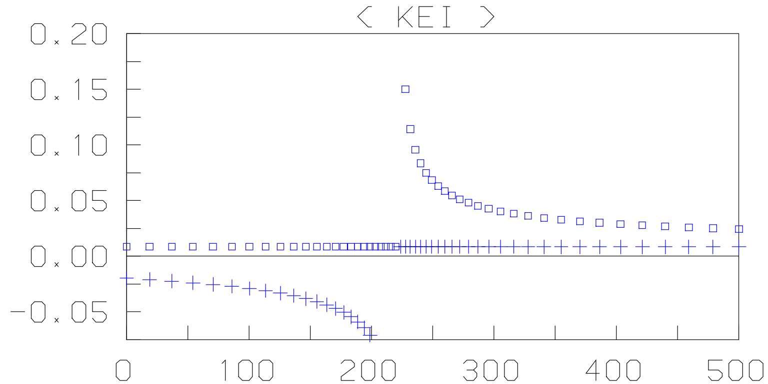
$$U_n(\xi) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \tau^{n-1}\, {\rm d}\tau\, \exp\left[-\frac{1}{2}\frac{\xi^2}{\tau^2} + {\rm i}\,\tau \right]$$

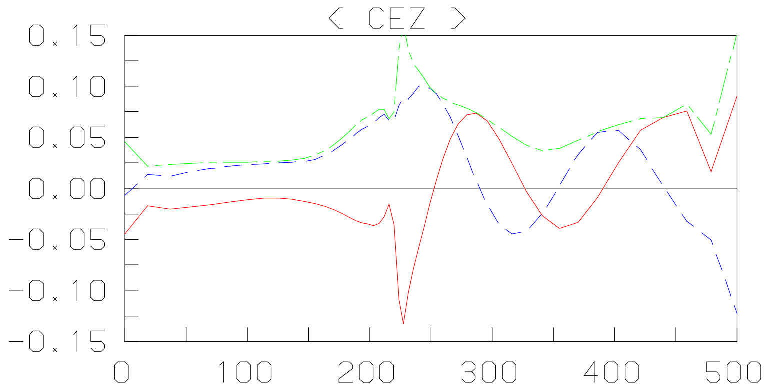
Linearly Polarized Wave: $\omega_{pe}^2/\omega^2 = 0.5$

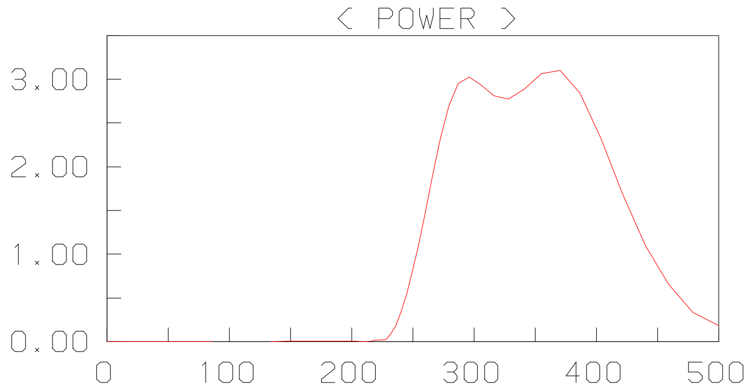


NZ	=	50	NW	=	50	
PNO	=	0.50000	ZMAX	=	500.0000	
PBO	=	0.90000	DZMAX	=	5.0000	
PB1	=	0.75150	DZWID	=	50.0000	
AKX	=	0.10000				
BETA	=	0.01000				
IN	=	0.0000	0.0000	1.0000	1.0000	
REFL	=	0.9515	0.0000	0.0003	0.0002	
P-IN	=	0.0000	0.0000	0.7509	0.2491	
P-REF	=	0.2424	0.0000	0.0002	0.0000	
P-ABS	=	0.7574	P-IN	-0.2424	0.9997	









Full Wave Analysis of Finite Gyro Radius Effects

- **Coordinates**
  - **Magnetic coordinate system**: (ψ, χ, ζ)
  - **Local Cartesian coordinate system**: (s, p, b)
  - **Fourier expansion**: poloidal and toroidal mode numbers, m, n
- **Perturbed current**

$$\boldsymbol{J}(\boldsymbol{r}, t) = -\frac{q}{m} \int \mathrm{d}\boldsymbol{v}\, q\boldsymbol{v} \int_{-\infty}^{\infty} \mathrm{d}t' \left[\boldsymbol{E}(\boldsymbol{r}', t') + \boldsymbol{v}' \times \boldsymbol{B}(\boldsymbol{r}', t')\right] \cdot \frac{\partial f_0(\boldsymbol{v}')}{\partial \boldsymbol{v}'}$$

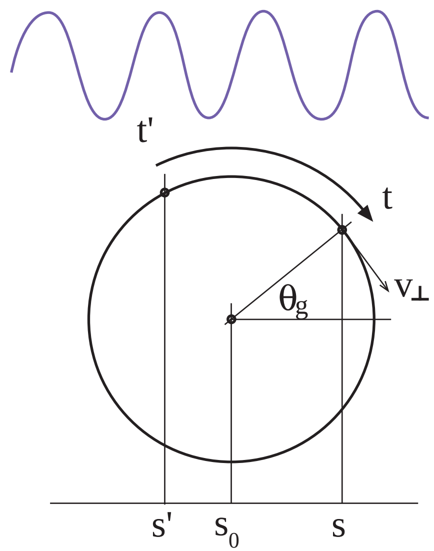
- **Maxwell distribution function**
  - Anisotropic Maxwell distribution with $T_\perp$ and $T_\parallel$:

$$f_0(s_0, \boldsymbol{v}) = n_0 \left(\frac{m}{2\pi T_\perp}\right)^{3/2} \left(\frac{T_\perp}{T_\parallel}\right)^{1/2} \exp\left[-\frac{v_\perp^2}{2v_{T_\perp}^2} - \frac{v_\parallel^2}{2v_{T_\parallel}^2}\right]$$

Variable Transformations

- **Transformation of Integral Variables**

  – Transformation from the velocity space variables $(v_\perp, \theta_g)$ to the particle position s' and the guiding center position s_0.



  – Jacobian: $J = \dfrac{\partial(v_\perp, \theta_g)}{\partial(s', s_0)} = -\dfrac{\omega_c^2}{v_\perp \sin \omega_c \tau}$.

  – Express $v_\perp$ and θ_g by s' and s_0 using $\tau = t - t'$, e.g.,

$$v_\perp \sin(\omega_c \tau + \theta_g) = \frac{\omega_c}{v_\perp} \frac{s - s'}{2} \frac{1}{\tan \frac{1}{2}\omega_c \tau} + \frac{\omega_c}{v_\perp} \left(\frac{s + s'}{2} - s_0 \right) \tan \frac{1}{2} \omega_c \tau$$

- **Integration over τ**: Fourier expansion with cyclotron motion

- **Integration over $v_\parallel$**: Plasma dispersion function

Final Form of Induced Current

- **Induced current**:

$$\cdot \begin{pmatrix} J_+^{mn}(s) \\ J_-^{mn}(s) \\ J_\parallel^{mn}(s) \end{pmatrix} = \int \mathrm{d}s' \sum_{m'n'} \overleftrightarrow{\sigma}^{m'n'mn}(s,s') \cdot \begin{pmatrix} E_+^{m'n'}(s') \\ E_-^{m'n'}(s') \\ E_\parallel^{m'n'}(s') \end{pmatrix}$$

- **Electrical conductivity**:

$$\overleftrightarrow{\sigma}^{m'n'mn}(s,s') = -in_0 \frac{q^2}{m} \sum_{\ell} \int \mathrm{d}s_0 \int_0^{2\pi} \mathrm{d}\chi_0 \int_0^{2\pi} \mathrm{d}\zeta_0 \exp i \left\{(m'-m)\chi_0 + (n'-n)\zeta_0\right\} \overleftrightarrow{H}_\ell(s,s',s_0,\chi_0,\zeta_0)$$

- **Matrix coefficients**: $\overleftrightarrow{H}_\ell(s,s',s_0,\chi_0,\zeta_0)$
  - Four kinds of **Kernel functions**
    - function of $s - s_0$, $s' - s_0$ and harmonics number ℓ
    - localized within several thermal gyroradii
    - depending on guiding center position (s_0, χ_0, ζ_0)
  - **Plasma dispersion function**

Kernel Functions

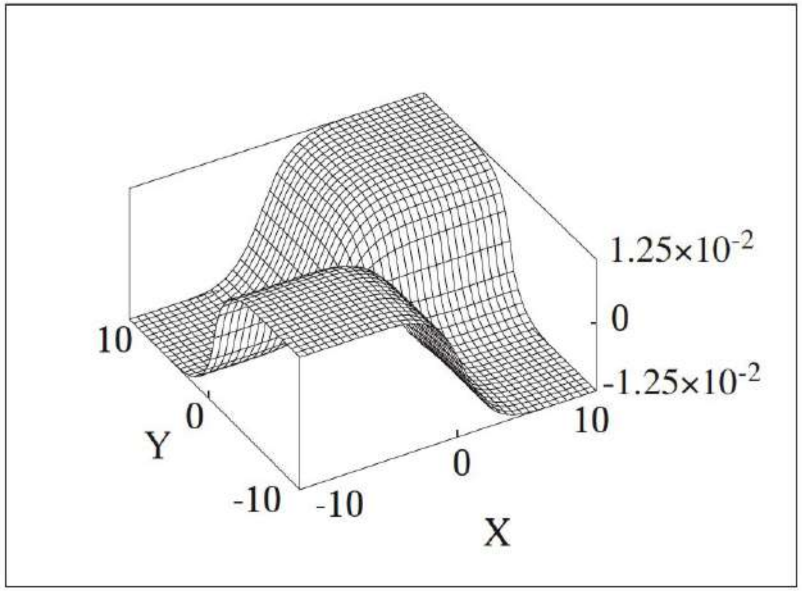
- **Kernel Function and its integrals**

$$F_n^{(i)}(X,Y) \equiv \frac{1}{2\pi^2}\int_0^{\pi} d\theta \exp\left[-\frac{X^2}{1+\cos\theta}-\frac{Y^2}{1-\cos\theta}\right] f_n^{(i)}(\theta)$$

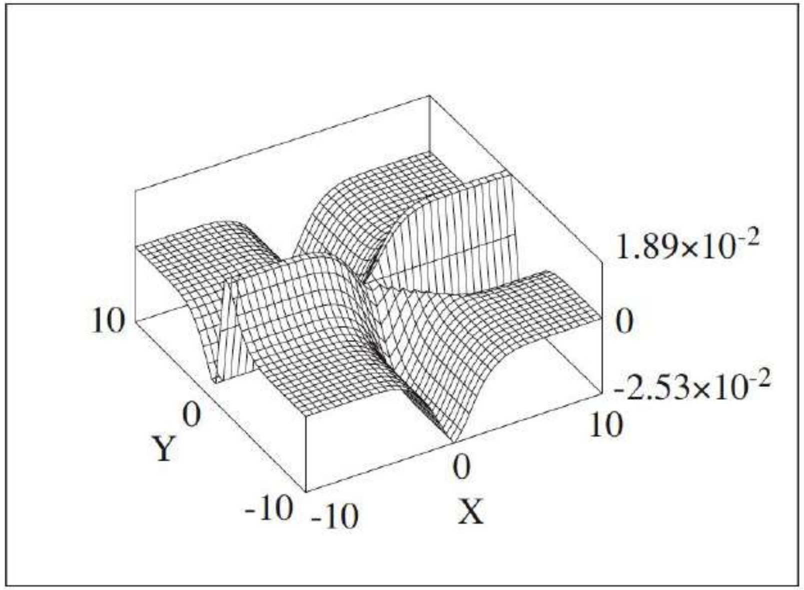
$$\mathcal{F}_n^{(ijk)}(X,Y) \equiv \int_0^{Y} dY' \int_0^{X+Y'} dX' X'^j Y'^k F_n^{(i)}(X',Y')$$

$$f_n^{(i)}(\theta) = \begin{cases} \dfrac{\cos n\theta}{\sin\theta} & (i=1) \\ \sin n\theta & (i=2) \\ \dfrac{\sin n\theta}{\sin^2\theta} & (i=3) \\ \dfrac{\cos\theta \sin n\theta}{\sin^2\theta} & (i=4) \end{cases}$$

$F_0^{(100)}$



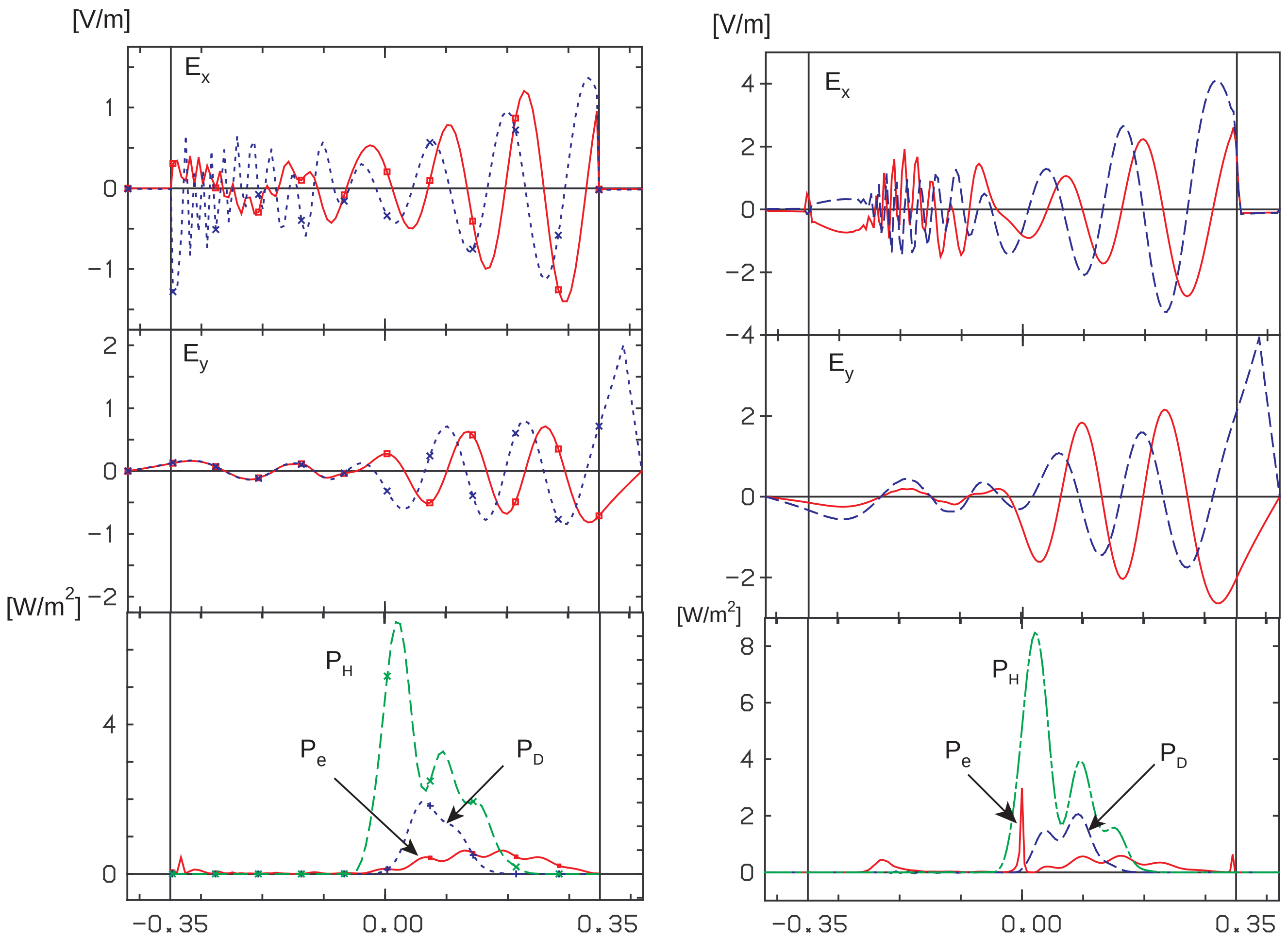
$F_1^{(100)}$



One-Dimensional Analysis (1)

ICRF minoring heating without energetic particles ($n_{\rm H}/n_{\rm D} = 0.1$)

Differential form　　　　Integral form



$$R_0 = 1.31\mathrm{m}$$

$$a = 0.35\mathrm{m}$$

$$B_0 = 1.4\mathrm{T}$$

$$T_{e0} = 1.5\mathrm{keV}$$

$$T_{D0} = 1.5\mathrm{keV}$$

$$T_{H0} = 1.5\mathrm{keV}$$

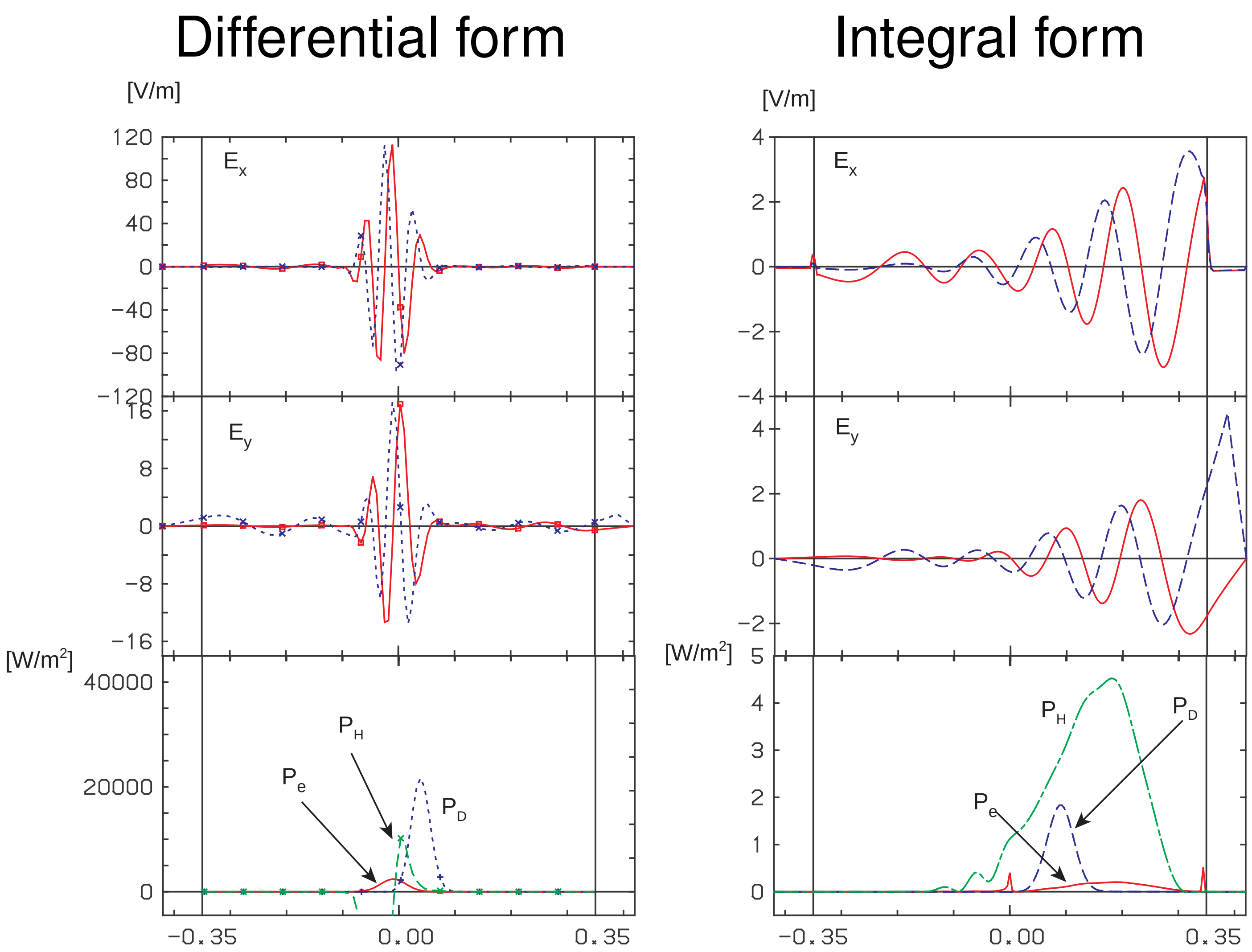
$$n_{s0} = 10^{20}\mathrm{m}^{-3}$$

$$\omega/2\pi = 20\mathrm{MHz}$$

Differential approach is applicable

One-Dimensional Analysis (2)

ICRF minoring heating with energetic particles $(n_{\rm H}/n_{\rm D} = 0.1)$



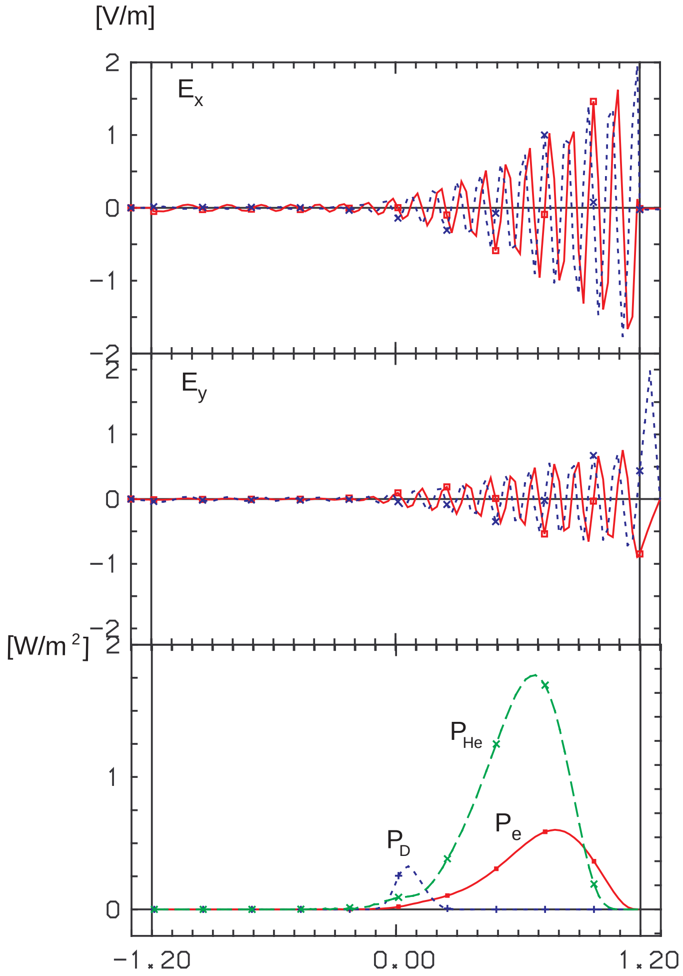
$$R_0 = 1.31\text{m}$$
$$a = 0.35\text{m}$$
$$B_0 = 1.4\text{T}$$
$$T_{e0} = 1\text{keV}$$
$$T_{\rm D0} = 1\text{keV}$$
$$T_{\rm H0} = 100\text{keV}$$
$$n_{s0} = 10^{20}\text{m}^{-3}$$
$$\omega/2\pi = 20\text{MHz}$$

Differential approach cannot be applied since $k_\perp \rho_i > 1$.

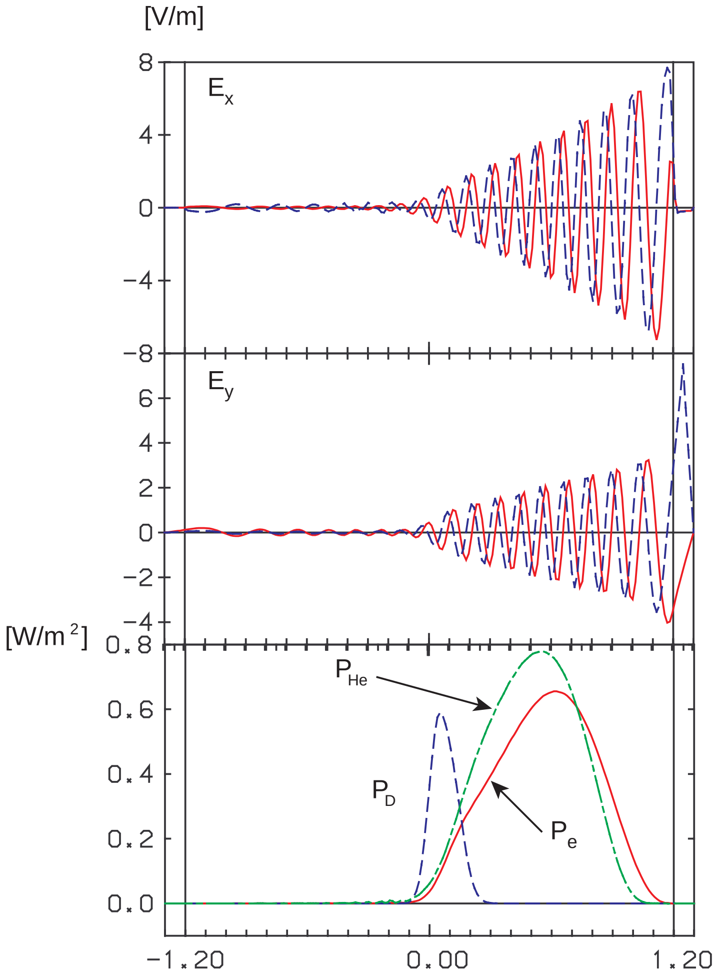
One-Dimensional Analysis (3)

ICRF minoring heating with α-particles ($n_\mathrm{D} : n_\mathrm{He} = 0.96 : 0.02$)

Differential form



Integral form



$$R_0 = 3.0\mathrm{m}$$
$$a = 1.2\mathrm{m}$$
$$B_0 = 3\mathrm{T}$$
$$T_{e0} = 10\mathrm{keV}$$
$$T_{D0} = 10\mathrm{keV}$$
$$T_{\alpha 0} = 3.5\mathrm{MeV}$$
$$n_{s0} = 10^{20}\mathrm{m}^{-3}$$
$$\omega/2\pi = 45\mathrm{MHz}$$

Absorption by α may be over- or under-estimated by differential approach.

Motivation of Kinetic Integrated Modeling

More accurate description of burning plasmas

- **Various heating mechanism: NB, IC, EC, LH, α particles**
  - Generation of energetic particles (EP)
  - Influence of EP on heating mechanism
  - Interaction of different heating schemes
- **Global instability: RWM, NTM, Internal kink, AE**
  - Influence of EP on global instabilities
  - Redistribution of EP due to global instabilities
- **Microscopic stabilities: ITG, TEM, KBM, ZF, GAM**
  - Influence of EP on turbulent transport
  - Influence of turbulence on EP transport

Issues in heating and current drive analysis

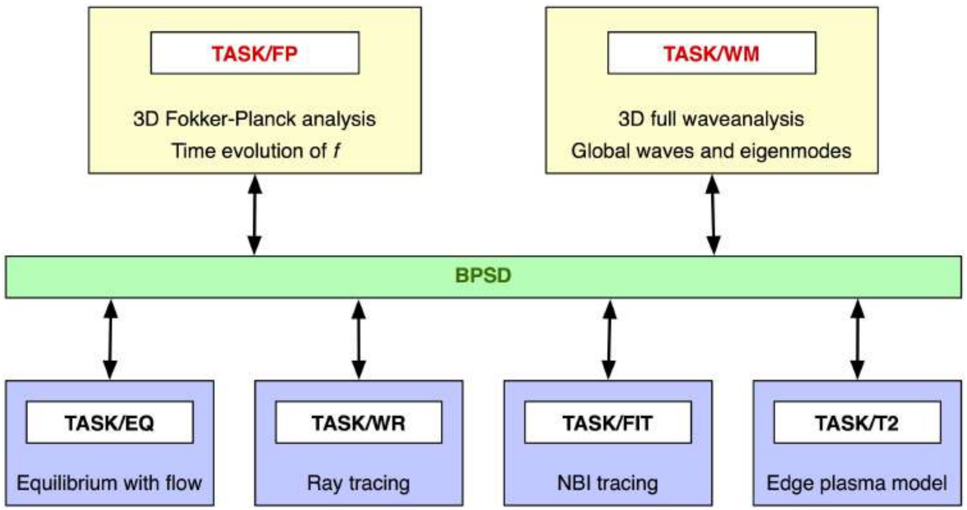
- **H/CD analysis in momentum space**
  - Usually for each species, sometimes for each heating scheme
  - Momentum is conserved, but energy is usually not, in FP
- **Inconsistency in FP-TR coupling**:

  Collisional heating power calculated by FP
  $\Longrightarrow$ Heat source in TR
  $\Longrightarrow$ Time evolution of temperature in TR
  $\Longrightarrow$ New bulk temperature to FP
  $\Longrightarrow$ Restart FP
- **Modification of $f(\boldsymbol{v}, \rho, t)$ due to radial transport**
  - Broadening of deposition profile
  - Spatial diffusion with energy dependence

Modeling based on momentum distribution function is required.

Kinetic Integrated Modeling: TASK3G

- **Integrated modeling based on momentum distribution function**
- **Two main components**
  - **Kinetic transport modeling**: Fokker-Planck analysis in 3D
    - Radial transport
    - Heating and current drive
  - **Kinetic full wave analysis**: Maxwell's equation
    - ICRF wave propagation and absorption
    - Global stability (Alfvén eigenmode, resistive wall mode, sawtooth)
- **Core components of TASK3G**



Kinetic Transport Modeling : TASK/FP

3D Fokker-Planck analysis

- **Bounce-averaged**:
  Trapped particle effect, zero banana width
- **Relativistic**:
  momentum p, relativistic collision operator
- **Non-Maxwellian collision operator**:
  momentum and energy conserved (2nd order Legendre exp.)
- **Multi-species**:
  conservation between species
- **Radial diffusion**:
  Anomalous radial diffusion with momentum dependence
- **Fusion reaction**:
  Contribution of energetic ions (integration in momentum space)
- **Parallel processing**:
  using PETSc library, one core one magnetic surface

Fokker-Planck Equation

- **Multi-species momentum distribution functions**:

$$f_s(p_{||}, p_\perp, \rho, t)$$

- **Fokker-Planck equation**

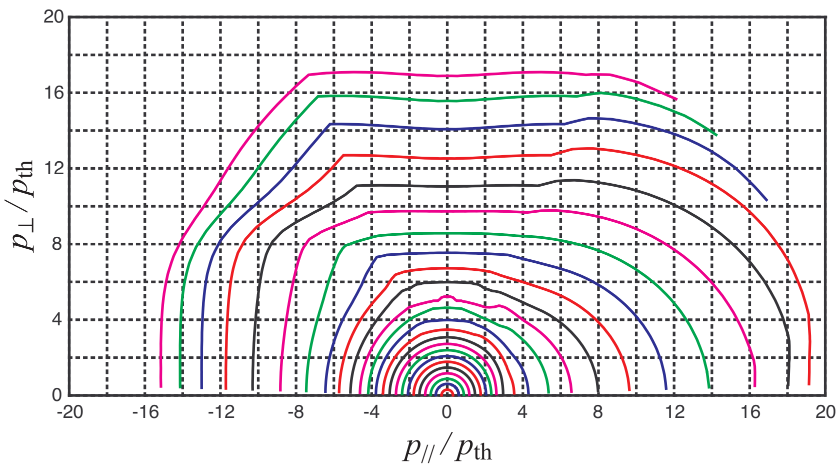
$$\frac{\partial f_s}{\partial t} = E(f_s) + C(f_s) + Q(f_s) + D(f_s) + S_s$$

  - $E(f)$: **Acceleration due to DC electric field**
  - $C(f)$: **Relativistic Non-Maxwellian Coulomb collision**
  - $Q(f)$: **Quasi-linear diffusion due to wave-particle resonance**
    - Full wave analysis (TASK/WM)
    - Ray/beam tracing (TASK/WR)
    - Fixed wave field profile; Fixed diffusion coefficient profile
  - $D(f)$: **Spatial diffusion**
  - S : **Particle Source and Sink** (NBI, Fusion reaction)

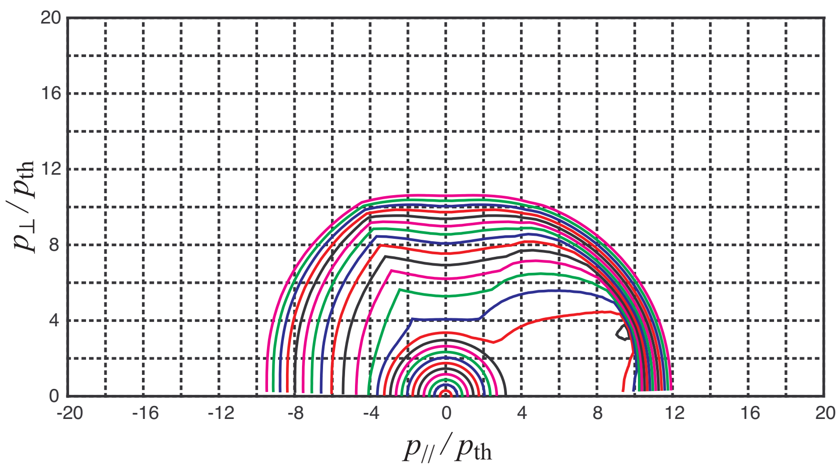
Multi-Species Fokker-Planck Analysis

Momentum distribution functions: $f_s(p_{||}, p_\perp, \rho, t)$

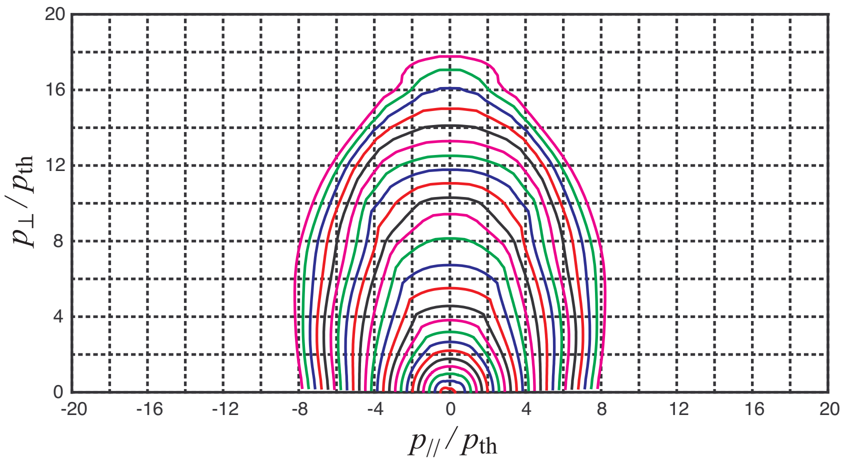
Electron : EC+LH



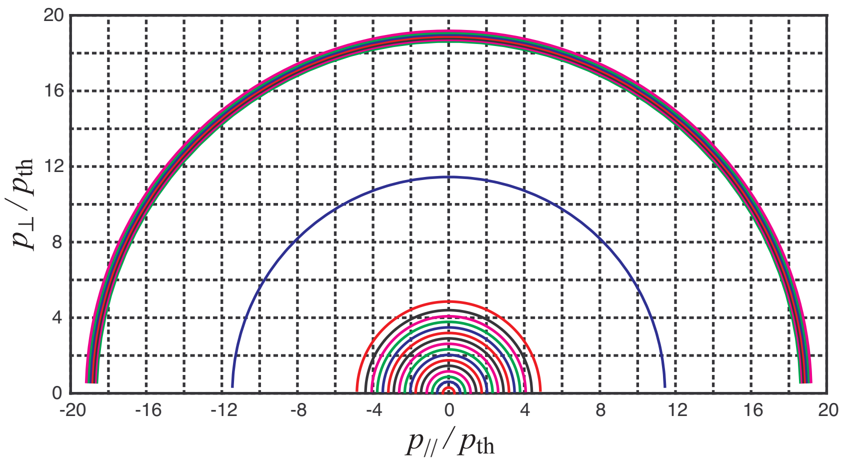
D : NBI



T : ICRF



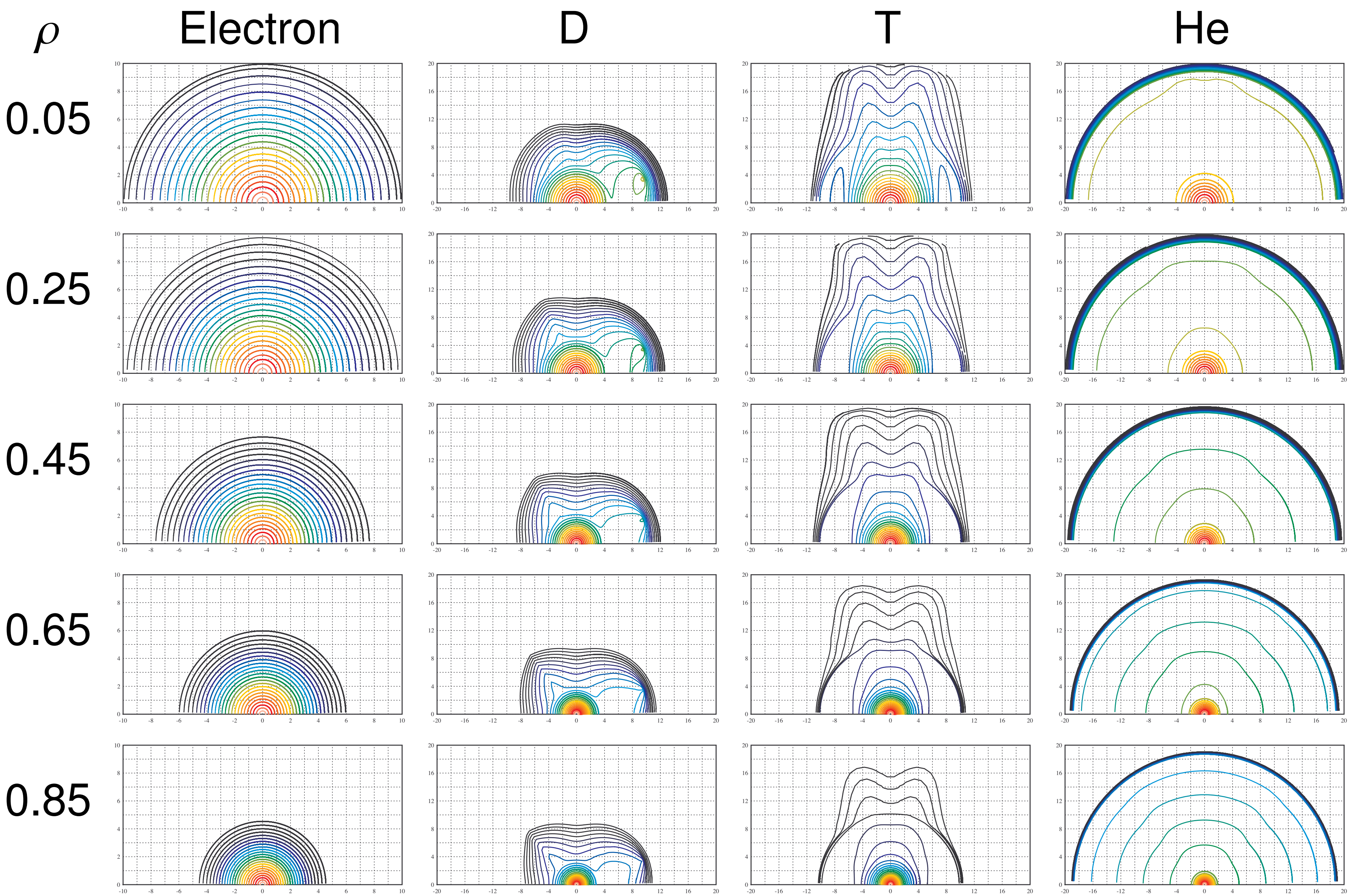
He : DT reaction



Analysis of Multi-Species Heating in ITER Plasma

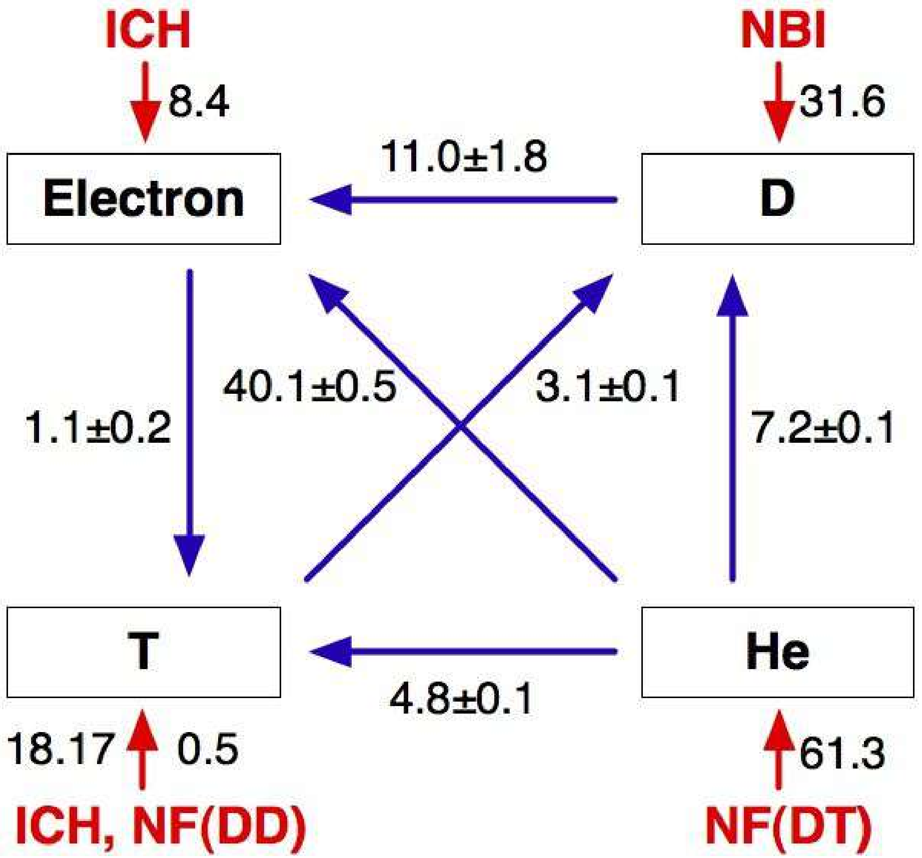
- **2D MHD equilibrium**
  - $R = 6.2\,\mathrm{m}$, $a = 2.0\,\mathrm{m}$, $\kappa = 1.7$, $\delta = 0.33$, $B_0 = 5.3\,\mathrm{T}$, $I_\mathrm{p} = 3\,\mathrm{MA}$
- **Multi species**:
  - Electron, D, T, He
- **Multi scheme heating**:
  - ICH, NBI, NF (DT, DD, TT)
- **Initial density**:
  - $n_\mathrm{e}(0) = 10^{20}\,\mathrm{m}^{-3}$, $n_\mathrm{D}(0) = 5 \times 10^{19}\,\mathrm{m}^{-3}$, $n_\mathrm{T}(0) = 5 \times 10^{19}\,\mathrm{m}^{-3}$
- **Initial temperature**:
  - $T_\mathrm{e}(0) = T_\mathrm{D}(0) = T_\mathrm{T}(0) = 20\,\mathrm{keV}$
- **Radial diffusion coefficient**: simplest model
  - $D_{rr} = 0.1(1 + 9\rho^2)\,\mathrm{m/s}$

Momentum Distribution Functions ($t = 1$ s)



Power Transfer between Species

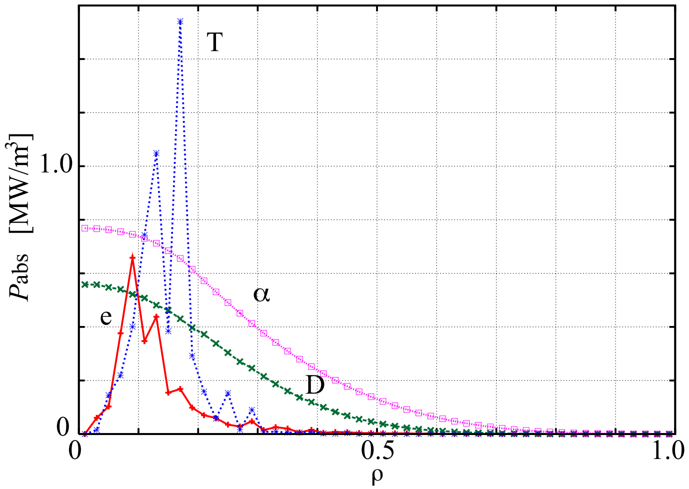
- **Collisional power transfer**



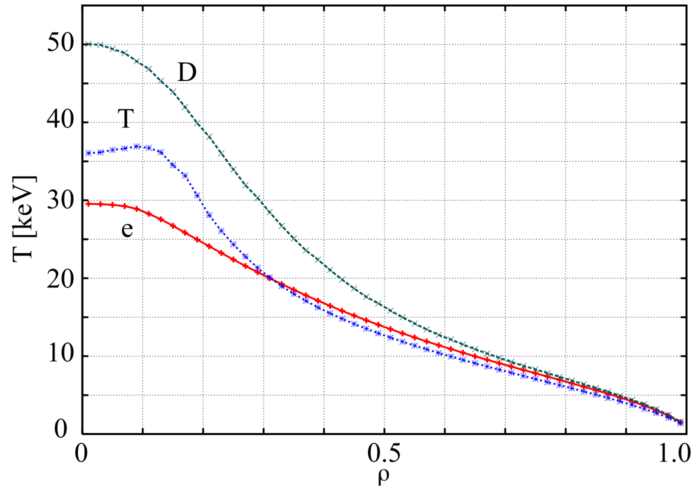
- **Requires more momentum meshes for better accuracy**
  - At present, typically $100 \times 100 \times 50$

Simulation with Radial Transport

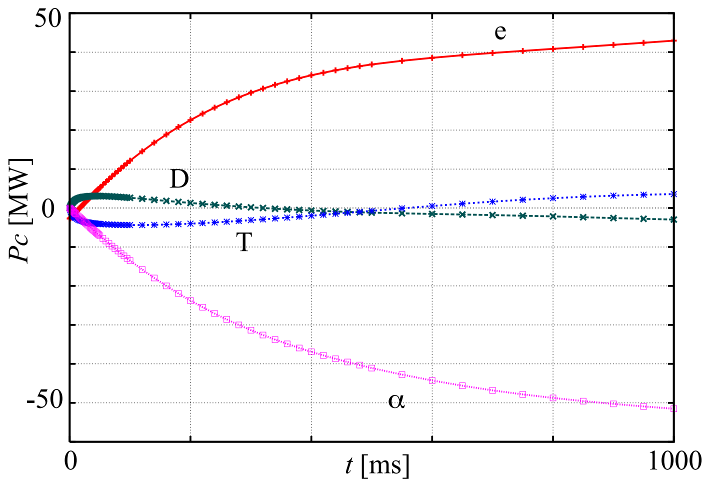
Absorbed power vs ρ



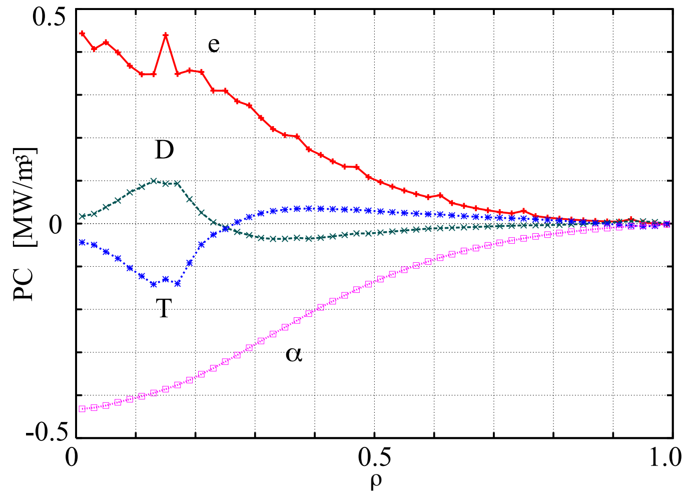
Kinetic energy density vs ρ



Collisional power transfer vs t



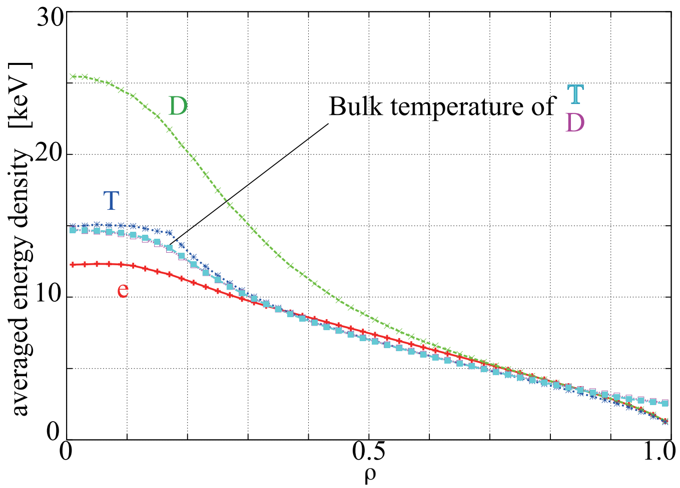
Collisional power transfer vs ρ



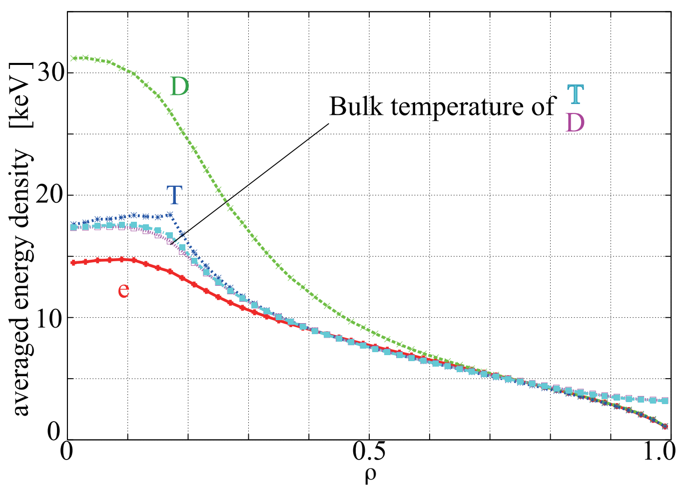
Dependence on Radial Diffusion Model

Radial profile of kinetic energy density: $p' = \sqrt{p^2 + p_{\mathrm{th0}}^2}$

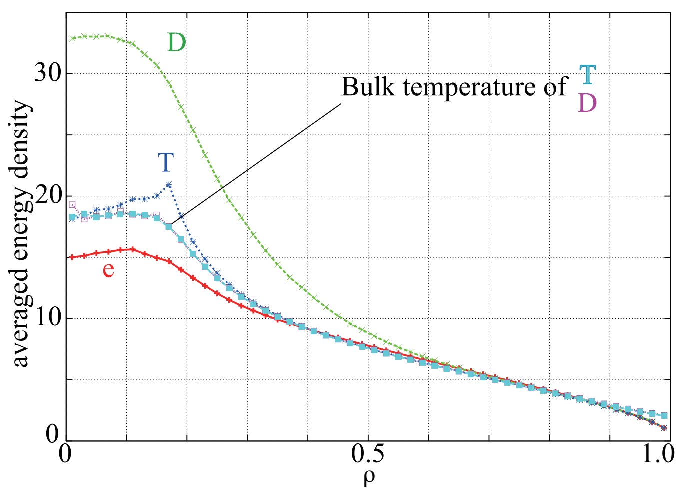
$D_{rr} \propto p'^0$



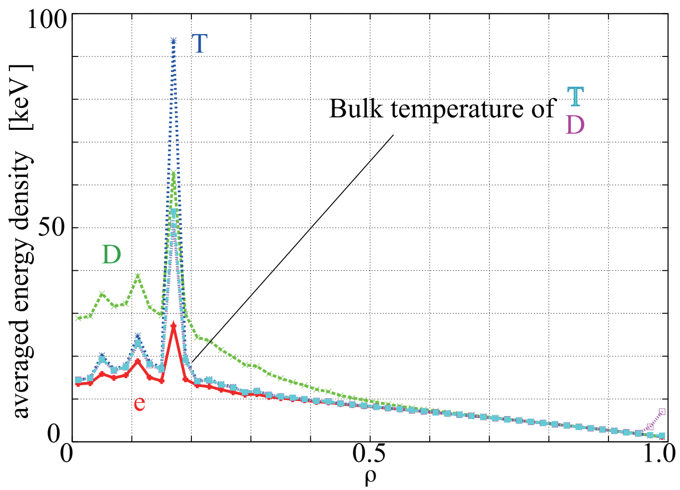
$D_{rr} \propto p'^{-1/2}$



$D_{rr} \propto p'^{-1}$



$D_{rr} = 0$



Dependence on Radial Diffusion Model

		$D_{rr} \propto p'^0$	$D_{rr} \propto p'^{-1/2}$	$D_{rr} \propto p'^{-1}$	$D_{rr} = 0$
E_{K} [keV]	e	7.13	7.63	7.74	8.18
	D	9.57	10.61	10.82	11.72
	T	7.18	8.00	8.15	9.44
	He	471.70	527.12	558.28	622.75
$T_{\mathrm{bulk,ave}}$ [keV]	e	7.16	7.64	7.80	8.18
	D	7.18	8.03	8.06	8.95
	T	7.13	7.98	7.98	8.87
	He	9.88	12.48	12.96	17.88
n_{ave} [$10^{16}\mathrm{m}^{-3}$]	He	6.45	8.36	8.64	9.65
P_{abs} [MW]	IC (e)	7.94	9.24	9.68	10.79
	IC (T)	8.95	10.34	10.87	15.28
	NB (D)	31.68	31.69	31.68	31.69
	$\mathrm{NF_{DT}}$	23.36	30.77	32.70	36.88
I_{CD} [MA]	I (e)	-1.13	-1.36	-1.44	-1.62
	I (D)	2.74	3.01	3.11	3.24
	T (tot)	1.61	1.65	1.67	1.62

Validation with experimental observation is necessary.

Issues in Kinetic Integrated Modeling

- **Modeling of transport process**
  - Turbulent transport coefficients with velocity dependence
  - Finite orbit size effects (Neoclassical transport)
  - Coupling with toroidal electric field (Faraday's law)
  - Keeping charge neutrality (Gauss's Law)
- **Kinetic full wave analysis**
  - Integral form of dielectric tensor including finite gyro radius effects
  - Gyrokinetic dielectric tensor for coupling with drift waves
- **Coupling with other components**
  - Equilibrium including kinetic effects (anisotropic pressure, and flow)
  - Modeling of diagnostics $\Longrightarrow$ **Validation by direct comparison**

Summary

- For better understanding of heating and current drive in magnetic fusion plasmas, **a kinetic integrated modeling code** is necessary.
- Our **kinetic integrated modeling code TASK** includes advanced wave analysis such as **beam tracing** and **2D FEM full wave analysis**, and implementation of **integral form of dielectric tensor** for kinetic full wave analysis is under way.
- The **multi-species Fokker-Planck component TASK/FP** enables us to simultaneously describe time evolution of 3D momentum distribution functions of electrons, deuterons, tritons and alpha particles in the presence of ECCD, ICRF heating, NBI heating, and fusion reactions, in ITER plasmas.
- **The momentum dependence of radial diffusion** strongly affects the formation on energetic ions and fusion reaction power.
- **Various issues still remain for kinetic integrated modeling.**